

**Matriculation Number:**

--	--	--	--	--	--	--	--	--	--

---

NATIONAL UNIVERSITY OF SINGAPORE

FACULTY OF SCIENCE

SEMESTER 1 EXAMINATION 2009-2010

**MA1505 MATHEMATICS I**

November 2009 Time allowed: 2 hours

---

**INSTRUCTIONS TO CANDIDATES**

1. **Write down your matriculation number neatly in the space provided above.** This booklet (and only this booklet) will be collected at the end of the examination. Do not insert any loose pages in the booklet.
  2. This examination paper consists of **EIGHT (8)** questions and comprises **THIRTY THREE (33)** printed pages.
  3. Answer **ALL** questions. For each question, write your answer in the box and your working in the space provided inside the booklet following that question.
  4. The marks for each question are indicated at the beginning of the question.
  5. Candidates may use calculators. However, they should lay out systematically the various steps in the calculations.
- 

**For official use only. Do not write below this line.**

Question	1	2	3	4	5	6	7	8
Marks								

**Question 1 (a)** [5 marks]

Find the slope of the tangent to the curve

$$y = (35x - 69)^{43}$$

when  $x = 2$ .

---

<b>Answer 1(a)</b>	
------------------------	--

*(Show your working below and on the next page.)*

*(More working space for Question 1(a))*

**Question 1 (b)** [5 marks]

Let

$$f(x) = ax^3 + bx^2$$

be a function defined on  $(-\infty, \infty)$ , where  $a$  and  $b$  are non-zero constants. Given that  $f$  has a point of inflection at  $(1, 2)$ , find the value of the product  $ab$ .

---

<b>Answer 1(b)</b>	
------------------------	--

*(Show your working below and on the next page.)*

*(More working space for Question 1(b))*

**Question 2 (a)** [5 marks]

Let

$$f(x) = \frac{23 - 4x}{7 - 2x}$$

and let

$$\sum_{n=0}^{\infty} c_n (x - 2)^n$$

be the Taylor series for  $f$  at  $x = 2$ . Find the **exact value** of  $c_0 + c_{2009}$ .

---

<b>Answer</b> <b>2(a)</b>	
------------------------------	--

*(Show your working below and on the next page.)*

*(More working space for Question 2(a))*

**Question 2 (b)** [5 marks]

Use the method of separation of variables to find  $u(x, y)$  that satisfies the partial differential equation

$$2u_{xy} = [\sin(x + y) + \sin(x - y)] u ,$$

given that  $u(0, 0) = 1$  and  $u(\pi, \pi) = e^2$ .

---

<b>Answer 2(b)</b>	
------------------------	--

*(Show your working below and on the next page.)*



*(More working space for Question 2(b))*

**Question 3 (a)** [5 marks]

Let

$$f(x) = x^2, \quad -\pi \leq x \leq \pi,$$

and  $f(x + 2\pi) = f(x)$  for all  $x$ . Let

$$a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

be the Fourier Series which represents  $f(x)$ . Find the **exact value** of  $a_{2010} + b_{2010}$ .

---

<b>Answer</b> <b>3(a)</b>	
------------------------------	--

*(Show your working below and on the next page.)*

*(More working space for Question 3(a))*

**Question 3 (b)** [5 marks]

Let

$$f(x) = \begin{cases} 1, & 0 < x < 1 \\ 2, & 1 < x < 2 \end{cases}.$$

Find the **exact expression** of the first two non-zero terms in the sine Fourier half range expansion for  $f(x)$ .

---

<b>Answer 3(b)</b>	
------------------------	--

*(Show your working below and on the next page.)*

*(More working space for Question 3(b))*

**Question 4 (a)** [5 marks]

Let  $S$  be the plane which passes through the points  $(1, 0, 0)$ ,  $(2, 1, 0)$  and  $(3, 2, 1)$ . Let  $L$  be the line which passes through  $(0, 0, 0)$  and is parallel to the vector  $-3\mathbf{i} + \mathbf{j} - \frac{26}{5}\mathbf{k}$ . Find the coordinates of the point of intersection of  $L$  and  $S$ .

---

<b>Answer 4(a)</b>	
------------------------	--

*(Show your working below and on the next page.)*

*(More working space for Question 4(a))*

**Question 4 (b)** [5 marks]

A space curve  $C$  is defined by the vector parametric equation

$$\mathbf{r}(t) = 2t^2\mathbf{i} + (t^2 - 4t)\mathbf{j} + (3t - 5)\mathbf{k}.$$

Let  $\mathbf{T}$  denote the tangent vector to  $C$  at the point corresponding to  $t = 1$ . Find the length of the projection of  $\mathbf{T}$  onto the vector  $\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ .

---

<b>Answer</b> <b>4(b)</b>	
------------------------------	--

*(Show your working below and on the next page.)*



*(More working space for Question 4(b))*

**Question 5 (a)** [5 marks]

Let

$$f(x, y, z) = xy + yz + zx + 1505.$$

Find the **exact value** of the directional derivative of  $f$  at the point  $(2, 3, 4)$  in the direction of the vector  $\mathbf{u} = \mathbf{i} - \mathbf{j} - \sqrt{2}\mathbf{k}$ .

---

<b>Answer</b> <b>5(a)</b>	
------------------------------	--

*(Show your working below and on the next page.)*

*(More working space for Question 5(a))*

**Question 5 (b)** [5 marks]

Find the local maximum points, local minimum points, and saddle points, if any, of the function

$$f(x, y) = xy + (x + y)(120 - x - y).$$

---

<b>Answer 5(b)</b>	
------------------------	--

*(Show your working below and on the next page.)*

*(More working space for Question 5(b))*

**Question 6 (a)** [5 marks]

Find the **exact value** of the double integral

$$\int \int_D \sqrt{|x - y|} dx dy,$$

where  $D$  is the rectangular region:  $0 \leq x \leq 1$  and  $0 \leq y \leq 2$ .

---

<b>Answer</b> <b>6(a)</b>	
------------------------------	--

*(Show your working below and on the next page.)*

*(More working space for Question 6(a))*

**Question 6 (b)** [5 marks]

Find the **exact value** of the iterated integral

$$\int_0^6 \left[ \int_x^6 \frac{2xy}{\ln\{(1+y^2)^{(1+x^2)}\}} dy \right] dx .$$

---

<b>Answer 6(b)</b>	
------------------------	--

*(Show your working below and on the next page.)*



*(More working space for Question 6(b))*

**Question 7 (a)** [5 marks]

Find the **exact value** of the volume of the solid enclosed laterally by the circular cylinder about  $z$ -axis of radius 1, bounded on top by the elliptic paraboloid

$$2x^2 + 4y^2 + z = 18 ,$$

and bounded below by the plane  $z = 0$ .

---

<b>Answer 7(a)</b>	
------------------------	--

*(Show your working below and on the next page.)*

*(More working space for Question 7(a))*

**Question 7 (b)** [5 marks]

Find the **exact value** of the line integral

$$\int_C (e^x \cos y) dx + (2x - e^x \sin y) dy ,$$

where  $C$  consists of two line segments:  $C_1$  from  $(\ln 3, 0)$  to  $(0, \frac{1}{\ln 36})$ , and  $C_2$  from  $(0, \frac{1}{\ln 36})$  to  $(-\ln 2, 0)$ .

---

<b>Answer</b> <b>7(b)</b>	
------------------------------	--

*(Show your working below and on the next page.)*

*(More working space for Question 7(b))*

**Question 8 (a)** [5 marks]

Find the **exact value** of the surface integral

$$\int \int_S z dS,$$

where  $S$  is the surface  $z = x^2 + y^2$  with  $0 \leq z \leq 1$ .

---

<b>Answer</b> <b>8(a)</b>	
------------------------------	--

*(Show your working below and on the next page.)*

*(More working space for Question 8(a))*

**Question 8 (b)** [5 marks]

Use Stokes' Theorem to find the **exact value** of the line integral

$$\oint_C (-yzdx + xzdy + xydz),$$

where  $C$  is the curve of intersection of the plane

$$x + y + z = 2$$

and the cylinder

$$x^2 + y^2 = 1,$$

oriented in the counterclockwise sense when viewed from above.

---

<b>Answer</b> <b>8(b)</b>	
------------------------------	--

*(Show your working below and on the next page.)*



*(More working space for Question 8(b))*

---

**END OF PAPER**