

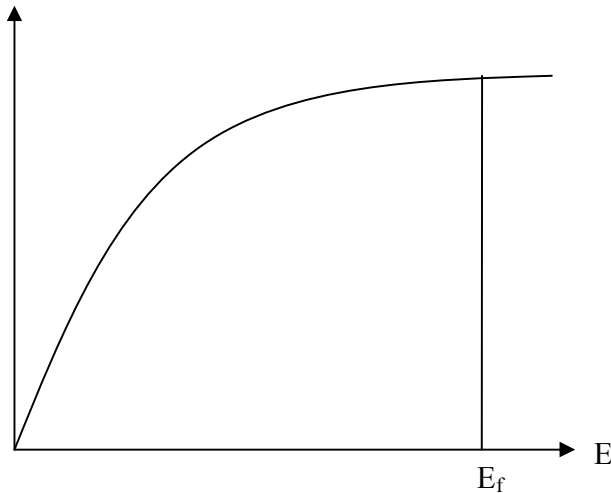
**PC3535 SOLID STATE PHYSICS I**  
**Nov 2003/04**

$$2(a) \quad N = 2 \frac{\frac{4}{3} \pi k^3}{\left(\frac{2\pi}{a}\right)^3} = \frac{a^3}{3\pi^2} \left(\frac{2mE}{\hbar^2}\right)^{\frac{3}{2}}$$

$$n = \frac{N}{V} = \frac{\frac{a^3}{3\pi^2} \left(\frac{2mE}{\hbar^2}\right)^{\frac{3}{2}}}{a^3} = \frac{1}{3\pi^2} \left(\frac{2mE}{\hbar^2}\right)^{\frac{3}{2}}$$

$$D(E) = \frac{dn(E)}{dE} = \frac{1}{2\pi^2} \left(\frac{2m}{\hbar^2}\right)^{\frac{3}{2}} E^{\frac{1}{2}}$$

As E approaches Fermi energy level, the density of state approaches a constant D(E)



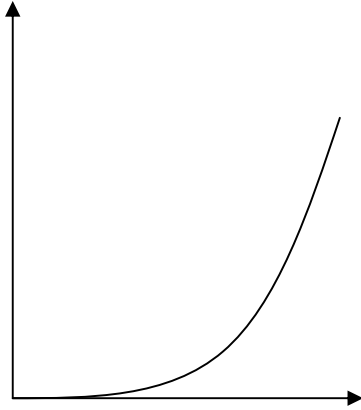
$$(b) \quad N = \frac{\frac{4}{3} \pi k^3}{\left(\frac{2\pi}{L}\right)^3} = \frac{L^3}{3(2\pi^2)} k^3 = \frac{V}{3(2\pi^2)} k^3$$

$$D(k) = \frac{dN}{dk} = \frac{V}{2\pi^2} k^2$$

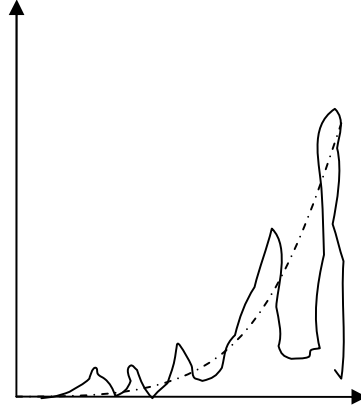
$$D(\omega)d\omega = D(k) \frac{dk}{d\omega} d\omega = \frac{D(k)}{v_g} d\omega = \frac{V}{2\pi^2 v_g} k^2 d\omega$$

Using Debye approximation,  $\omega = vk$

$$D(\omega) = \frac{V}{2\pi^2 v_g} \left( \frac{\omega}{v_g} \right)^2 = \frac{V \omega^2}{2\pi^2 v_g^3}$$



(a)



(b)

As a Debby solid, the density of states is a smooth curve (Fig a), however in reality, this is not true. Instead the density of states fluctuates above and below the curve denoted by the Debby solid (Fig b). However, this jagged curve can be approximated to the Debby solid as the total area under the curves is the same for both graphs as explained by Debby.

(c) Since we know  $\omega(k)$ , as before,  $D(\omega) = D(k) \frac{dk}{d\omega}$

$$D(k) = \frac{dN}{dk} = \frac{V}{2\pi} k^2 \Rightarrow D(\omega) = \frac{V}{2\pi} k^2 \frac{dk}{d\omega}$$

3. (a)  $J = \sigma E = -nev$

$$\therefore \sigma = -\frac{nev}{E} \quad \dots(1)$$

$$F = m \frac{dv}{dt} = -eE \Rightarrow m \int_0^v dv = -eE \int_0^\tau dt$$

$$v = -\frac{eE\tau}{m} \quad \dots(2)$$

$$\sigma = -\frac{ne}{E} \left( -\frac{eE\tau}{m} \right) = \frac{ne^2\tau}{m}$$

Electrical resistivity of most metals at room temperature is higher than at liquid helium temperature. At room temperature, the movement of electrons is impeded by other electrons as well as the vibration of the ions within the crystal lattice. Therefore the relaxation time is very short. In contrast, the movement of conduction electrons at liquid helium temperature is also impeded by other electrons as well as the vibration of the ions within the crystal lattice. However,

due to the low temperature, the probability of conduction electrons being liberated from the valence band is significantly lesser as governed by the Fermi probability function. Therefore, the relaxation time is longer than that at room temperature.

$$(b) \quad m \left( \frac{d}{dt} + \frac{1}{\tau} \right) v = -eE$$

$$\text{Let } v = v_0 e^{-i\omega t}, \quad \frac{dv}{dt} = -i\omega v,$$

$$m \left( \frac{d}{dt} + \frac{1}{\tau} \right) v = m \left( \frac{dv}{dt} + \frac{v}{\tau} \right) = m \left( \frac{-i\omega\tau + 1}{\tau} \right) v = -eE$$

$$\text{Since } J = \sigma E = -nev, \quad v = -\frac{\sigma E}{ne}$$

$$\left( -\frac{\sigma E}{ne} \right) m \left( \frac{-i\omega\tau + 1}{\tau} \right) = -eE$$

$$\sigma = \frac{ne^2\tau}{m} \left( \frac{1}{1 - i\omega\tau} \right) = \frac{ne^2\tau}{m} \left( \frac{1 + i\omega\tau}{1 + (\omega\tau)^2} \right)$$

The real part represents the actual conductivity of the system while the imaginary part describes the attenuation of conductivity.

$$(c) \quad \text{Lorentz force, } F = -(e\mathbf{E} + e\mathbf{v} \times \mathbf{B})$$

$$\text{Let } \mathbf{B} = (0 \quad 0 \quad B) \text{ and } \mathbf{J} = \left( \frac{ne^2\tau}{m} E_x \quad 0 \quad 0 \right)$$

$$\mathbf{F} = -e \left( \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} + \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ v_x & v_y & v_z \\ 0 & 0 & B \end{vmatrix} \right) = -e \left( \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} + \begin{pmatrix} Bv_y \\ -Bv_x \\ 0 \end{pmatrix} \right) = m \left( \frac{dv}{dt} + \frac{v}{\tau} \right)$$

$$\text{At steady state, } \frac{dv_x}{dt} = \frac{dv_y}{dt} = \frac{dv_z}{dt} = 0$$

$$\therefore \frac{m}{\tau} \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = -e \begin{pmatrix} E_x + Bv_y \\ E_y - Bv_x \\ E_z \end{pmatrix}$$

$$\frac{m}{\tau} v_x = -e(E_x + Bv_y), \quad \frac{m}{\tau} v_y = -e(E_y - Bv_x), \quad \frac{m}{\tau} v_z = -eE_z$$

$$\text{Let } v_y = 0, \quad 0 = -e(E_y - Bv_x)$$

$$v_x = \frac{E_y}{B} \Rightarrow \frac{m}{\tau} v_x = -e(E_x + Bv_y) = \frac{m}{\tau} \left( \frac{E_y}{B} \right)$$

$$R_H = \frac{E_y}{J_x B} = \frac{-\frac{e\tau E_x B}{m}}{\frac{ne^2\tau E_x B}{m}} = -\frac{1}{ne}$$

From the formula derived,  $R_H = -\frac{1}{ne}$ , we can see that the Hall coefficient is inversely proportional to the concentration of electrons. As metals have a very high concentration of electrons,  $n$  will be large. This would imply that the hall coefficient will be small thus it is very hard to measure Hall coefficient in metals.

$$4(a) \quad a_{As} = \frac{m}{m^*} \left( \frac{\epsilon_s}{\epsilon_0} \right) (0.529)$$

$$\frac{E_{As}}{E_h} = \frac{m}{m^*} \left( \frac{\epsilon_0}{\epsilon_s} \right)^2 \Rightarrow \frac{m}{m^*} = \frac{E_{As}}{E_h} \left( \frac{\epsilon_s}{\epsilon_0} \right)^2$$

$$\therefore a_{As} = \frac{E_{As}}{E_h} \left( \frac{\epsilon_s}{\epsilon_0} \right)^3 (0.529) \approx 1581 \text{ \AA}$$

$$N_{enclosed} = 2.0 \times 10^{23} \times (10^{-2})^{-3} \times \frac{4}{3} \pi (1581 \times 10^{-10})^3 \approx 3.31 \times 10^9 \text{ electrons}$$

(b) (i)

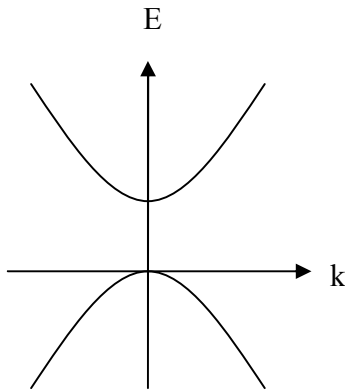


Fig a

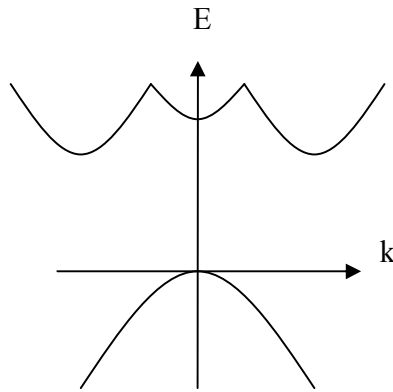


Fig b

Direct bandgap as illustrated by Fig a has the lowest conduction band directly above the peak of the valence band. This implies that momentum is conserved if it energy is to be imputed into a transiting electron.

Indirect band gap occurs when the lowest value of the conduction band is not at the same wave vector as that of the peak of the valence band. This implies that if a transiting electron may not have enough momentum to transit between the bands if only a phonon of energy level equals to the energy gap is absorbed

(ii)  $E = \hbar\omega \Rightarrow \omega = \frac{E}{\hbar}$

$$p = \hbar k = \frac{E}{c}; \text{ as } \omega = ck \text{ for photon}$$

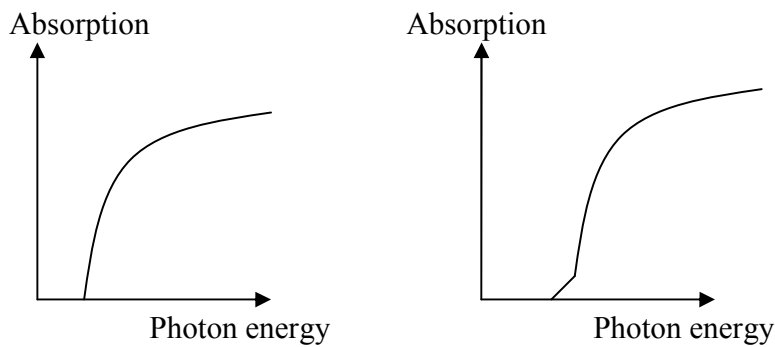
$$\approx 3.74 \times 10^{-28} \text{ kg m s}^{-1}$$

for electron,

$$p = \hbar k = \hbar \left( \frac{2\pi}{a} \right) \approx 1.17 \times 10^{-24}$$

the momentum required to transit from the valance band to the conduction band in an indirect band gap is 4 orders of magnitude more than the incoming photon can provide. Therefore such a transition will not occur if only a photon is there to supply the energy for transition.

- (c) If direct band gap were to occur, experimental results will yield a clean curve as seen in the left Figure.



If an indirect band gap were to happen, experimental results will yield a graph similar to that on the right. This is because, the photons are not absorbed directly, instead an additional absorption of a phonon is required for a transition between valance band and conduction band to occur.