National University of Singapore

PC3235 Solid State Physics I

(Semester I: AY2011-12, 28 November 2011)

Time Allowed: Two Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **THREE** questions and comprises **FOUR** printed pages.
2. Answer **ALL THREE** questions.
3. Answers to the questions are to be written in the answer books.
4. This is a CLOSED BOOK examination.
5. A Table of Constants is provided.

1. (a) (i) Calculate the density of electrons (n) in a conventional unit cell of a Si crystal with lattice parameter of $5.4A$.

[3 marks]

(ii) Derive an expression for the Fermi momentum in terms of n, based on the free electron model.

[3 marks]

(b) Describe briefly experiments to measure the following parameters of a p-type silicon sample: (i) confirm the sign of the majority carrier; (ii) majority carrier concentration; (iii) band gap; (iv) effective masses; (v) mobility of the majority carrier.

[9 marks]

(c) For a pure semiconductor, the expressions for concentration of carriers are given as below:

$$n = 2\left(\frac{m_e k_B T}{2\pi\hbar^2}\right)^{\frac{3}{2}} exp\left[\frac{(E_F - E_c)}{k_B T}\right] \qquad (1)$$

$$p = 2\left(\frac{m_h k_B T}{2\pi\hbar^2}\right)^{\frac{3}{2}} exp\left[\frac{(E_v - E_F)}{k_B T}\right] \qquad (2)$$

Where E_F is the Fermi level; E_v, E_c the energy at the valence band edge and conduction band edge respectively; m_h, m_e the effective mass of hole and electron respectively.

(i) Show that the Fermi level in an intrinsic semiconductor lies in the middle of the gap at low temperature. State clearly the condition for it to happen.

[5 marks]

(ii) Calculate the hole density at room temperature for a given direct gap doped semiconductor. The electron density is $10^{23}/m^3$, the gap is $1.0eV$ and the pre-exponential factors in equation (1) and (2) are 1.1×10^{25} and $0.51 \times 10^{25} m^{-3}$ respectively at room temperature.

[5 marks]

2. (a) (i) Derive an expression for density of states for frequencies ω, $D(\omega)$, for an one-dimensional chain of atoms with the dispersion relation of $\omega = \omega_m \left|\sin\frac{Ka}{2}\right|$, where ω_m is a constant.

[6 marks]

(ii) Sketch the $D(\omega)$ vs ω graph and comment on its behavior.

[2 marks]

(iii) Sketch Debye density of states in the same graph of part (ii) and indicate the cut-off frequency clearly.

[4 marks]

(b) Give a simple and approximate qualitative expression for the heat capacity C_v of Fermi gas with N electrons at low temperature.

[4 marks]

(c) The experimental heat capacity expression for a metal at low temperature is given by

$$\frac{C_v}{T} = (2.1 + 2.6T^2)\ mJ\ mol^{-1}K^{-1} \qquad (3)$$

(i) Explain the origin of the two terms in the expression.

[4 marks]

(ii) Calculate the Fermi energy for the metal.

[5 marks]

3. (a) Various statements of the Bragg condition are:

$$2d \sin \theta = n\lambda \qquad (4)$$

$$\Delta\vec{K} = \vec{G} \qquad (5)$$

$$2\vec{K} \cdot \vec{G} = G^2 \qquad (6)$$

Where $\vec{K} + \Delta\vec{K} = \vec{K}'$, $\vec{K}$ is the incident wave vector, $\vec{K}'$ the outgoing vector and $\vec{G}$ is the reciprocal lattice vector.

(i) Show that equation (4) and equation (6) are equivalent.

[4 marks]

(ii) Show that equation (5) and equation (6) are equivalent.

[3 marks]

(iii) Use equation (6) to show that the reflection of electron waves in one dimensional lattice occurs at $K = \pm\frac{\pi}{a}$.

[3 marks]

(b) Using band folding concept, explain how a free electron energy vs wave vector relation in the reduced zone scheme can be constructed.

[4 marks]

(c) Sketch a free electron Fermi surface of occupied states in square lattice using perturbation by a crystal potential model for the following two cases.

(i) Lattice with one electron per primitive cell.

[3 marks]

(ii) Divalent metal.

[3 marks]

Indicate the zone boundary clearly in your sketch.

(iii) Hence, explain why a divalent metal is a conductor.

[5 marks]

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