PC4199 Honours Project in Physics: COUNTS IN CELL ANALYSIS OF COSMIC MICROWAVE BACKGROUND RADIATION

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2 ABSTRACT

This project is a computational based project on the Planck mission's archived cosmic microwave background radiation data.This article discuss analysis of Planck space mission composite Cosmic Microwave Background(CMB) anisotropy Map. The article study hotspot and coldspot in cosmic microwave background map and discuss the causes of these spots. With the main contributor to anisotropies as adiabatic fluctuations, the CMB is strongly related to galactic formation. These allows for comparison with theoretical model galaxy clustering in gravitational Quasi-Equilibrium distribution. Utilising c program language, a composite map from Planck mission is analysed. The counts in cell distribution of the number of hotspot is tabulated with the program. The results show that anisotropy of cosmic microwave background agrees with the Gravitational Quasi-Equilibrium distribution.

3 INTRODUCTION

The paper will begin with introduction to the basic term and theoretical knowledge. This involves a short introduction to Big Bang cosmological model and cosmic microwave background radiation(CMB). The next few sections discuss causes of the anisotropy in cosmic microwave background with method to formulate partition function and distribution function. This conclude the theoretical background and the next section(DATA) highlights the sample and the strategy for analysis of the sample. The program description and explanation is discussed in the COUNTS IN CELL STRATEGY section. This is followed by the results and discussion in the next two section(RESULTS , DISCUSSION AND CONCLUSION). Finally, appendix A contains the program code used in the project followed by the bibliography.

3.1 BIG BANG COSMOLOGY

To understand the focus of this project, there is a need to look at the origins of cosmic microwave background radiation. Thus the project will begin by reviewing Big Bang cosmological model and understand how the CMB was produced. Big Bang cosmological model is a widely accepted model for the origin of the universe. Big Bang model explains that the universe began with a singularity at around 13.7 billion years ago. This early universe is superheated and extremely dense. The universe then expands and cools rapidly in a process known as inflation. As the rapid expansion ceased, the universe consist of quark-gluon plasma and elementary particles. The temperature of the universe was high and the particles are to be treated as relativistic particles. Particles-antiparticles pair were continuously created and destroyed. Baryogenesis occurs which led to an excess of quarks and leptons to their anti-particle counterpart. The universe continues to expand slowly with symmetry breaking for the fundamental forces as the universe cools. The temperature is no longer able to support particle-antiparticle pair creation and mass annihilation occurs. However due to baryogensis, there will be an excess of elementary particles like proton, neutron and electrons. Big bang nucleosynthesis, the process where proton bond with neutron to form deuterium and helium occurs at temperature around a billion kelvin. This period of time is known as radiation dominated era. Thermalization occurs during this period with bremsstrahlung (free-free scattering) and Thompson scattering in the plasma like fluid which results in the temperature of radiation across the universe to be uniform. Also, radiation dominated universe is like an opaque fog which prevent photon from travelling freely. This is due to the dense universe with many free particles serving as scattering centres for radiation. The photons, free proton and electrons interact through Compton and Thompson scattering which causes the photon to be unable to travel freely.

As the universe continues to expand and cools. Most of the free particles like electrons, neutron and proton have been removed by nucleosynthesis. This period of time is known as recombination epoch or surface of last scattering. As the free particles that serve as scattering centres are no longer present, radiation can travel freely in the universe. This is the period where matter decouples from radiation. This happens at 3000 kelvin and redshift of decoupling is around 1089. These first thermal radiations to travel the universe freely are the cosmic microwave background radiation.

Subsequently, the universe continues to cools and expands while the particles start to cluster together due to gravity. These particles come together to form star, galaxies and other astronomical structures.

3.2 COSMIC MICROWAVE BACKGROUND RADIATION

Cosmic Microwave Background Radiation also known as CMB are leftover thermal radiation from the period of last scattering in Big Bang Cosmology. CMB is the first photon to travel in the universe after the surface of last scattering. Cosmic microwave background radiation were once high energy thermal radiation but has lost most of its energy as it travels freely through the universe. Cosmic microwave background radiation were first detected by Arno Penzias and Robert Wilson. They operated a 20 foot horn shaped reflector antenna and pointed it in various directions and acertain the presence of a isotropic radiation not from any specific galactic source. The detector at that time would have measured the cosmic microwave background radiation as a blackbody radiation of 3.5 kelvin. Recent satellite measurements of cosmic microwave background radiation shows a reading of 2.7 Kelvin and cosmic microwave background radiation will continue to be redshifted as it travel through the universe. Cosmic microwave background radiation is thought to be isotropy and homogeneous. However with more sensitive detectors, tiny anisotropies were found across cosmic microwave background radiation. There have been many studies on these anisotropies as they offer insights to the structures of the early universe, age of universe and the universe contents.

Anisotropies in cosmic microwave background radiation is usually characterised by the CMB power spectrum in figure below.

Anisotropies in cosmic microwave background radiation can be represented with spherical harmonies in a power spectrum. The power spectrum represent a mean to display temperature fluctuation with multipoles. Multipoles represent the number of fluctuations or modes around the sky. The temperature fluctuation can be projected in a 2D spherical surface and represented with spherical harmonics:

$$Y_{lm} = \sqrt[\text{ffl}]{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}}P_l^m(cos\theta)e^i m\phi \tag{1}$$

where the indices are $l = 0, ..., \infty$ and $-l \leq m \leq l$ and P_l^m are the legendre polynomials. l is known as multipole and this multipole can be related to angular scale in the sky by $\theta = \pi/l$. This angle θ is the typical size of a cycle of fluctuation for that specific multipole function. The multipole is a representation of the number of waves along a meridian with the waves as anisotropy in temperature fluctuations of CMB.

$$T(\hat{n}) = \sum_{l=0}^{l=\infty}\sum_{m=-l}^{l} \alpha_{lm}Y_{lm}(\hat{n}) \tag{2}$$

where

$$\alpha_{lm} = \int_{\theta=-\pi}^{\pi}\int_{\phi=0}^{2\pi} T(\hat{n})Y_{lm}^*(\hat{n})d\omega \tag{3}$$

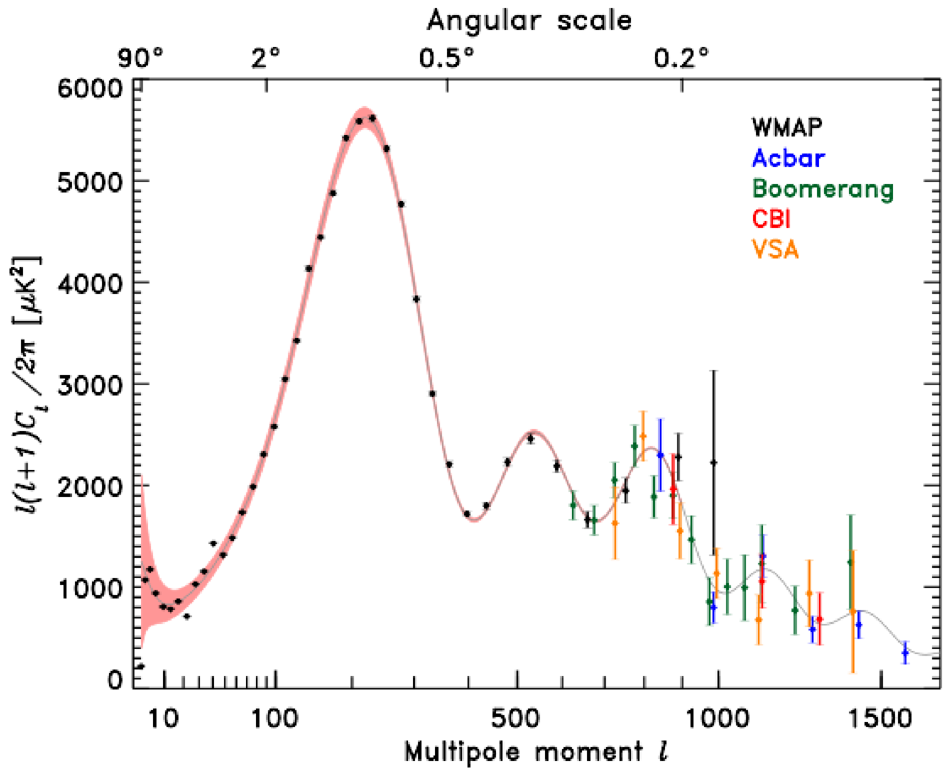

Figure 1: Power spectrum for anisotropy in Cosmic Microwave Background Radiation. Image from: http://www.universeadventure.org/big$_b$*ang/popups/cmb − dtrh − powerspectrum.htm*

$$C_l = \frac{1}{2l+1} \sum_{m=-l}^{l} \langle |\alpha_{lm}|^2 \rangle \tag{4}$$

$$\langle |\alpha_{lm} \alpha^*_{lm}| \rangle = \delta_{lm} \delta_{l'm'} C_l \tag{5}$$

The power spectrum characterised the amount of fluctuation in cosmic microwave background radiation temperature spectrum at different angular scales across the sky. The angular power spectrum breaks down the fluctuations into a series of multipoles functions. The power spectrum(figure 1) measures the temperature variation in the y axis and the multipoles on the x-axis. The power spectrum shape can be used to determine the oscillations of the hot gas in the early universe and contents of the early universe. This power spectrum will be related to causes of anisotropy in cosmic microwave background radiation in the next section.

4 CAUSES OF ANISOTROPIES IN COSMIC MICROWAVE BACKGROUND RADIATION

Anisotropies in Cosmic Microwave background radiation is usually classified by the period in which it occurs as primary and secondary causes. Primary causes occur before the surface of last scattering and recombination epoch while secondary causes occur after the surface of last scattering. Primary causes include Sachs-Wolfe effect, adiabatic fluctuations while secondary causes include Early and Late integrated Sachs-Wolfe effect and Sunyaez-Zel'dovich effect.

Adiabatic fluctuations occur within the baryon and photon plasma before the surface of last scattering. Before the surface of last scattering, the plasma with elementary particles may contain slight mass perturbations across the plasma. In adiabatic fluctuation, a denser region is hotter and will emit photon with higher energy. This leads to photons of these regions possessing a higher blackbody temperature. At the surface of last scattering, recombination occurs between the free elementary particles. That photon decouples from the denser mass region and were released as cosmic microwave background radiation. These photons previously coupled with the denser mass region will appear to have more energy compared to cosmic microwave background radiation from other regions. As a result anisotropies will appear throughout the cosmic microwave background radiation due to adiabatic fluctuations.

Sachs-Wolfe effect is gravitational red-shift of the cosmic microwave background radiation. Before surface of last scattering, the universe exists in a state of plasma with baryons and photons. This plasma acts like a fluid that leads to mass fluctuations within the plasma. This mass fluctuation causes regions of non-zero gravitational potential to appear. As the universe is still expanding, density fluctuation will be changed across the universe. This leads to change in gravitational potential across these density fluctuations. A photon travelling through a gravitational created by mass fluctuation, will experience an increase in net energy as it travels through a gravitational well that is gradually smoothed out by expansion. This is a decrease in gravitational potential due to expanding universe. By theory of relativity, photons travelling through these changing gravitational potential will experience a net gravitational blueshift or redshift depending on the change. A decreasing gravitational potential leads to blueshift while an increasing gravitational potential result in gravitational redshift.

Primary causes of anisotropy in cosmic microwave background radiation originate from the mass fluctuations of the fluid like plasma form in the early universe. This mass fluctuation will proceed to grow denser with gravitational clustering and proceed to form galaxies as the universe expands. Hence a way to study the cosmic microwave background radiation is to compare distribution function for gravitational clustering to study anisotropies in cosmic microwave background.

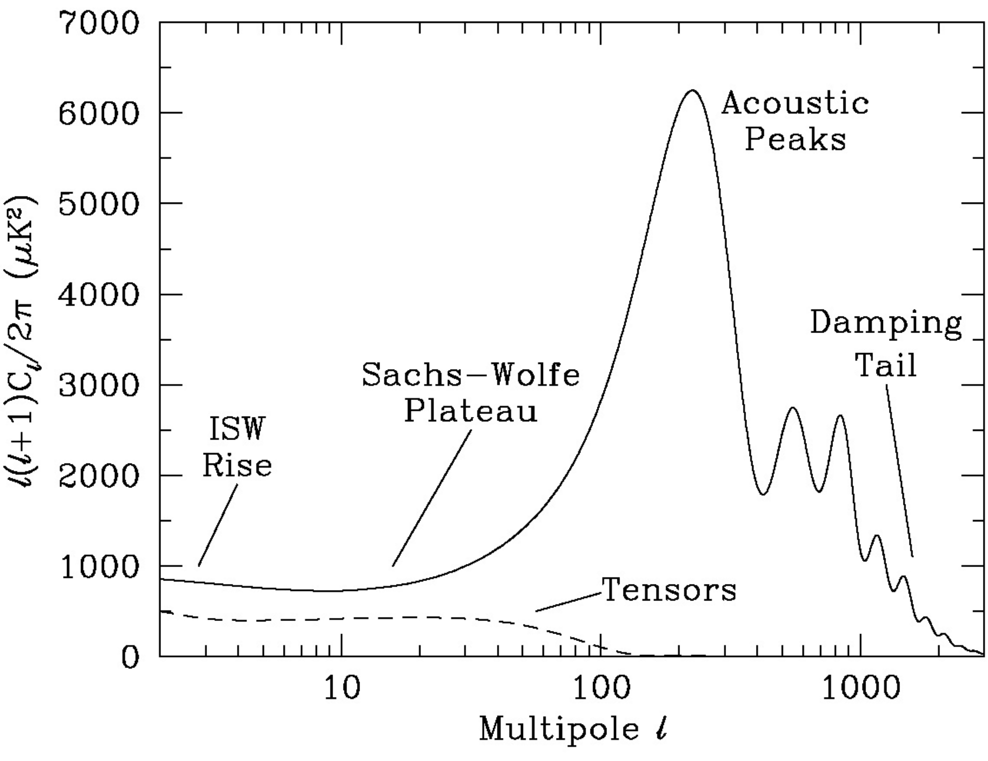

Figure 2: Power spectrum for anisotropy in CMB. Image from: https://ned.ipac.caltech.edu/level5/March05/Scott/Scott4.html

Early and late integrated Sachs-Wolfe effect is gravitational redshift of the cosmic microwave background radiation. This occurs as a combination of effects form an expanding universe and gravitational well. Gravitational redshift was predicted by Einstein in 1911, radiation travelling form regions of stronger gravitational field to area of lower gravitational field will experience a reduction in energy and the radiation experience redshift. While radiation will experience blue shift if it travel from an area with weaker gravitational field to a stronger gravitational field. These occur at galactic supercluster and voids. Normally radiation passing through a galactic supercluster will experience equal amount of blue shift and redshift. However, as the universe expands, the density of the supercluster is reduced and the gravitational potential well is reduced. The radiation will be redshifted to a lesser degree compared to blue shifted as the gravitational potential has been reduced. The radiation will gain a net increase in energy as it travels through a supercluster and the reverse occurs for voids. These effects can be combined with the previous effects as they have a similar impact on cosmic microwave background and occur at the same region.

Sunyaez-Zel'dovich effect occurs with high energy electron supply energy to cosmic microwave background radiation through Inverse Compton scattering. In dense galaxies, there are clusters of free high energy electrons that can acts as high energy scattering centres for radiation. When low energy cosmic microwave background radiation scatters of these high energy electrons, it results in an increase in energy of the cosmic microwave background radiation. The cosmic microwave background radiation experiences blue shift and will appear to be hotter.

Secondary causes specifically Sunyaez-Zel'dovich effect originates from large scale structures and galaxies which are most prevalent in the galactic plane. From the power spectrum, the different causes are prevalent at different angular angles(figure 2). Sunyaez-Zel'dovich effect[28] are prevalent at lower angular angles (> 1000) as shown in the power spectrum figure. Thus Sunyaez-Zel'dovich effect can be minimised by restricting angular scale in this project. An angular scale of $1°$ (around $l = 180$) will suffice for measuring the effects of the causes mentioned before (figure 2). In this project, primary causes will be the main focus and will be analysed with distribution function based on gravitational clustering as both originates from density fluctuation in the early universe

5 PLANCK MISSION

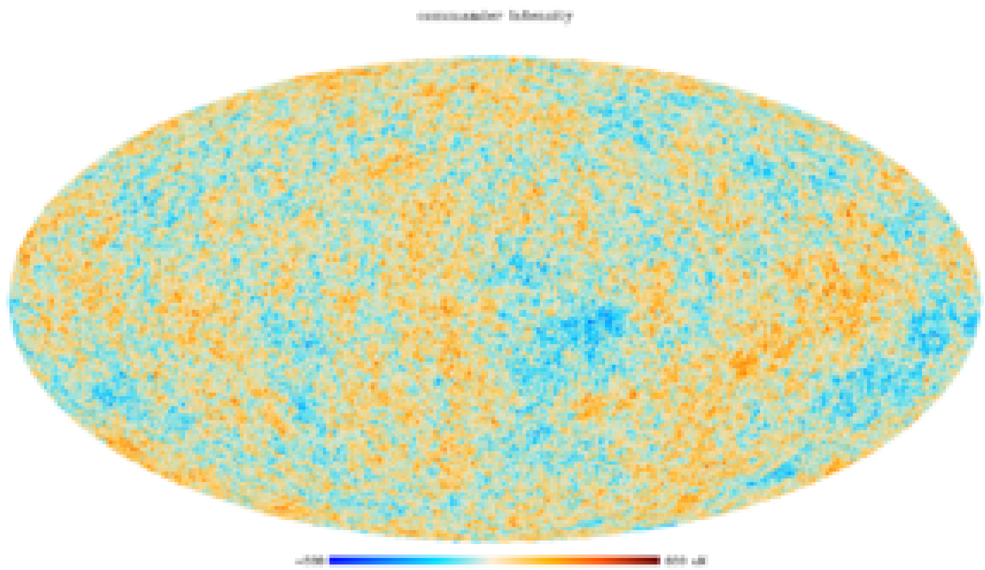

Figure 3: Cosmic Microwave Background Radiation composite map [24]

Planck Mission is a space observatory launched by European Space agency in 2009 to 2013. The mission collected data of cosmic microwave background radiation in the the microwave and infra-red wavelength. The mission utilised bolometers to measure cosmic microwave background radiation. The mission has higher sensitivity than previous space observatory for CMB like WMAP and COBE. This allows us to study anisotropy in cosmic microwave background radiation with greater accuracies than ever before. The Plank mission have a variety of aims including high resolution detection of both intensity and polarization of primordial cosmic microwave background radiation anisotropies, catalogue of galaxy clusters through sunyaez Zel'dovich effect and gravitational lensing of CMB. Planck space observatory took measurement at Earth/Sun L_2 point. The space observatory took two all sky survey by 2013 with bolometers[24]. Bolometers are detectors that measure the power of an incident radiation by measuring the change in resistance of a temperature dependent material. It consists of a thermal reservoir, absorptive material and a resistivity detector. The thermal reservoir maintains the temperature and determines the sensitivity of the device. When the absorptive material absorbs electromagnetic radiation, temperature of the material is raised and the change in resistivity of the material is recorded. These bolometers are designed to be sensitive to a specific bandwidth and the result tabulated. The results will be used to plot the black-body radiation curve and temperature of cosmic microwave background radiation is determined. The Planck spacecraft moved along the ecliptic plane at one degree per day and spun with one revolution per minute. The Planck mission spacecraft have a field of view around 85 degree centred on the Ecliptic plane. The Planck mission data are published online and can be accessed through the European Space agency website[20].

Figure 3 shows the composite map of anisotropy in cosmic microwave background. The figure has been processed and averaged on 2.7 kelvin. The range of the composite map is between -30 micro kelvins and 30 micro kelvins. This composite map will be used in this project and discussed in the DATA section.

6 PARTITION FUNCTION OF COSMIC MICROWAVE BACKGROUND RADIATION

To better understand and analyse the results gotten from the Planck mission, there is a need to construct a partition function for the cause of cosmic microwave background radiation. Partition function of cosmic microwave background radiation can be worked from many angles[22]. It is difficult to formulate a partition function for the photons due to the largely different temperature and changing number of photons. One way to bypass this difficulty and facilitate analysis is to formulate the partition function based on factors directly related to the anisotropy of cosmic microwave background radiation. In this project, the partition function is produced directly from the causes of anisotropy in cosmic microwave background radiation. From the previous section on causes of anisotropies in cosmic microwave background radiation, Primary causes are deemed as the primary source of anisotropies. Hence the partition function will be constructed based on adiabatic fluctuations and Sachs-Wolfe effect. The partition function will be based on the mass fluctuations responsible for anisotropy in cosmic microwave background radiation. The mass are coupled with cosmic microwave background radiation right before the surface of last scattering. The cosmic microwave background radiation emitted by the mass in the early universe and will reflect the temperature of these mass. In an adiabatic system, the denser regions will have higher temperature. These denser regions in the plasma medium will causes hotter cosmic microwave background radiation and vice versa. Hence the partition function of the masses in the early universe will suffice as the partition function of the cosmic microwave background radiation. The use of this partition function further facilitates analysis of the cosmic microwave background radiation with that of gravitational clustering in the universe. As the hotter and denser regions in the plasma medium responsible for anisotropy will act as centre for gravitational clustering. These clusters of mass will continue to grow by gravitational clustering as the universe aged. They will serve as early galaxies seedling and grow to be the structures in the current observable galaxy. Hence both anisotropies in CMB and gravitational clustering originate from the same source.

The project utilise Gravitational Quasi-Equilibrium distribution (GQED)[5] in the formulation of the partition function. Gravitational Quasi-Equilibrium distribution assumes that gravitating clustering evolves through a series of quasi stable states. GQED is a thermodynamic description with a parameter b that is a clustering parameter. This parameter b is a ratio of gravitational correlation energy to twice the kinetic energy of peculiar velocities relative to Hubble flow. Distribution function of GQED[1] is given by:

$$f(N) = \bar{N}(1 - b_\epsilon)[\bar{N}(1 - b_\epsilon) + Nb_\epsilon]^{N-1}e^{-\bar{N}(1-b_\epsilon)-Nb_\epsilon} \tag{6}$$

b can be related to the variance of the counts-in-cells distribution:

$$\langle (\Delta N)^2 \rangle = \frac{\bar{N}}{(1-b)^2} \tag{7}$$

b can also be related to the two point correlation function:

$$b = 1 - (\bar{N}\epsilon_2(V) + 1) \tag{8}$$

The equations above will be derived in the following sections and these three equations are fundamental equations used to analyse the data gotten from the composite map. To begin constructing a partition function of gravitating mass, the classical discrete system is used. At the surface of last scattering, the temperature of the universe is estimated to be around 3000K. This can be estimated by solving a particular case of saha ionization equation to obtain an equation relating ratio of electron number density and total baryon number density to temperature[25].

$$\frac{{N_e}^2}{N_B} = (\frac{m_e kT}{2\pi\bar{h}^2})^{\frac{3}{2}} exp(\frac{-B}{kT}) \tag{9}$$

where B is binding energy which is 13.59 eV for hydrogen, N_e stand for electron number density and N_B is the total baryon number density.

$$\frac{x^2}{1-x} = \frac{1}{N_B}(\frac{m_e kT}{2\pi\bar{h}^2})^{\frac{3}{2}} exp(\frac{-B}{kT}), \tag{10}$$

where x is $x = \frac{N_e}{N_B}$ At the recombination epoch, the free electron number drop drastically and is reflected in the equation. Value of x drops drastically to near zero at temperature range of 5000K to 2500K. Combined with parameter of the current observable universe, the temperature is estimated at 3000K for $x = 0.003$.[25] The partition function will be developed based on the mass right after the surface of last scattering. At this stage, the free particles will be no longer present and the universe will no longer be coupled to the cosmic microwave background radiation. The early universe resemble that of a gas with hydrogen, deuterium, helium and dark matter. With the kinetic theory for gas, the root mean square speed of the particles can be determined.

$$KE = \frac{3}{2}KT \quad (11)$$

$$\frac{1}{2}mv^2 = \frac{3}{2}KT \quad (12)$$

$$v = \sqrt[2]{\frac{3KT}{m}} \quad (13)$$

$$v = \sqrt[2]{\frac{3(1.38 * 10^{-23})(3000K)}{1.660 * 10^{-27}}} \quad (14)$$

The root mean square speed is $v = 8649m/s = 0.002c$ which is a fraction of the speed of light. Thus the partition function will be based on classical discrete system. The partition will then be constructed with gravitational energy and kinetic energy in a classical discrete system. The following section is a re-derivation of the partition function from Ahmad, Saslaw and Bhat statistical mechanics of the cosmological many body problem[1]. The classical partition function will be formulated with N particles of mass M interacting gravitationally with potential energy ϕ and have momentum p_i at average temperature T:

$$Z_N(T,N) = \frac{1}{N!\lambda^{3N}} \int exp-[\sum_{i=1}^{N} \frac{{p_i}^2}{2m} + \phi(r_1, r_2, ..., r_N)]T^{-1}d^{3N}pd^{3N}r \quad (15)$$

N! represent the distinguishability of classical particles and λ normalizes the phase space volume cell. Similar to a partition function of perfect classical gas, the partition function is split into kinetic energy and gravitational energy part.

$$Z_N(T,N) = \frac{1}{N!\lambda^{3N}} \int exp(-[\sum_{i=1}^{N} \frac{{p_i}^2}{2m})exp(\phi(r_1, r_2, ..., r_N)]T^{-1})d^{3N}pd^{3N}r \quad (16)$$

Where λ normalizes the phase space volume cell. The kinetic energy closely resembles that of perfect classical gas[27]. With Gaussian integral of form:

$$\int exp(-\alpha x^2)dx = \sqrt{\frac{\pi}{\alpha}} \quad (17)$$

Thus the kinetic energy part of the partition function can be resolved to with gaussian integral by consider a single particle partition function initially[27] :

$$Z_N(T,N) = (2\pi mT)^{\frac{3}{2}} \quad (18)$$

Incorporate into the general partition function with N particles:

$$Z_N(T,N) = \frac{1}{N!}(\frac{2\pi mT}{\lambda^2})^{\frac{3N}{2}} Q_N(T,V) \tag{19}$$

where $Q_N(T,V)$ is the configuration integral

$$Q_N(T,V) = \int ... \int \prod_{1\leq i\leq j\leq N} exp[-\phi_{ij(r)}T^{-1}]d^{3N}r \tag{20}$$

The configuration integral is difficult to evaluate and usually involves a virial expansion with powers of density. However for cosmological gravitational many body problems, the configuration integral can be simplified without virial expansion. The gravitational potential energy is based with the relative position as gravitational potential can be treated as pair interactions. The gravitational potential energy can be rewritten as a function of relative position vector between any two particles.

$$\phi(r_1, r_2, ..., r_N) = \sum_1 \phi(r_{ij}) = \sum_1 \phi_{ij}(r) \tag{21}$$

The configuration integral will be:

$$Q_N(T,V) = \int ... \int \prod_{1\leq i\leq j\leq N} exp[-\phi_{ij(r)}T^{-1}]d^{3N}r \tag{22}$$

Next f_{ij} is introduced as the two-particle function which:

$$f_{ij} = e^{\frac{-\phi_{ij}}{T}} - 1 \tag{23}$$

f_{ij} is zero if there is no interaction between particles like ideal gas and at high temperature f_{ij} will be close to zero. Incorporating the two-particle function into the configuration integral:

$$Q_N(T,V) = \int ... \int \prod_{1\leq i\leq j\leq N} [1+f_{ij}]d^3r_1 d^3r_2 ... d^3r_N \tag{24}$$

As the particles are considered as interacting gravitationally in the configuration integral. The integral can be simplified by dropping higher order term since gravitating system can be resolved to pair interactions and self energy terms f_{jj} can be dropped The product in the configuration integral will be rewritten as:

$$\prod_{1\leq i\leq j\leq N}(1+f_{ij}) = \prod_{1\leq i}\prod_{i\leq j\leq N}(1+f_{ij}) = \prod_{j=2,3,4,...N}(1+f_{1j})(1+f_{2j})...(1+f_{Nj}) \tag{25}$$

Thus the configuration integral will becomes:

$$Q_N(T,V) = \int ... \int \prod_{1\leq i\leq j\leq N} (1+f_{12})(1+f_{13})(1+f_{23})(1+f_{14})...(1+f_{N-1,N})d^3r_1d^3r_2...d^3r_N \tag{26}$$

However, this representation of gravitational potential diverges when r_{ij} approaches zero for point mass approximation. This can fixed by introduced a softening parameter to take into account the finite size of the particle. The softening parameter describe the finite size of galaxy in constant proper coordinates and is between 0.01 to 0.05. The gravitational potential energy can be rewritten as:

$$\phi_{ij} = \frac{-Gm^2}{(r_ij^2+\epsilon^2)^{\frac{1}{2}}} \tag{27}$$

The two particle interaction function will become:

$$1+f_{ij} = exp[\frac{Gm^2}{T({r_{ij}}^2+\epsilon^2)^{\frac{1}{2}}}] \tag{28}$$

We treat the system as undervirialzed system which refers to a system that is still clustering. In a undervirialized system, $\frac{Gm^2}{Tr_{ij}^{\frac{1}{2}}}$ can be taken as small and the two point interaction can be approximated by removing higher powers. For undervirialized system, the equation can be approximated with Taylor series to:

$$f_{ij} = \frac{Gm^2}{T({r_{ij}}^2+\epsilon^2)^{\frac{1}{2}}} \tag{29}$$

At this stage, the configuration integral can be evaluated for different values of N and generalized. For N = 1:

$$Q_1(T,V) = V \tag{30}$$

For N = 2:

$$Q_2(T,V) = \int\int [1+\frac{Gm^2}{T({r_{ij}}^2+\epsilon^2)^{\frac{1}{2}}}]d^3r_1d^3r_2 \tag{31}$$

To evaluate the integral for higher order of N, the position of one particle can be fixed and the rest of the integration is done relative to this fixed particle. Furthermore the expansion of the universe exactly cancels the long range mean gravitational field on the particles' motion. Thus integration of the integral is only up to an average scale R_1. Beyond R_1 density fluctuations do not diverge much from the average density. This integration is only over a region of limited region of space where f_{ij} is appreciable.

$$Q_2(T,V) = \int_0^{2\pi}\int_0^{\pi}\int_0^{R}[1+\frac{Gm^2}{T({r_2}^2+\epsilon^2)^{\frac{1}{2}}}]d^3r_1 r_2^2 sin(\theta)d^3r_1 dr_2 d\theta d\phi \tag{32}$$

The first volume integral gives V while the second volume integral can be solved with the following integral:

$$\int\frac{x^2}{\sqrt[2]{x^2+a^2}}dx = \frac{1}{2}x\sqrt[2]{x^2+a^2}+\frac{1}{2}a^2 ln|x+\sqrt[2]{x^2+a^2}| \tag{33}$$

$$Q_2(T,V) = 4\pi V[\frac{R_1^3}{3}+\frac{Gm^2}{T}(\frac{1}{2}R_1^2\sqrt[2]{1+\frac{\epsilon^2}{R_1^2}}+\frac{1}{2}\epsilon^2 ln|R_1+R_1\sqrt[2]{1+\frac{\epsilon^2}{R_1^2}}|)] \tag{34}$$

$$Q_2(T,V) = V^2[1+\frac{3Gm^2}{2T}(\frac{1}{R_1}\sqrt[2]{1+\frac{\epsilon^2}{R_1^2}}+\frac{1}{R_1}\frac{\epsilon^2}{R_1^2}ln|R_1+R_1\sqrt[2]{1+\frac{\epsilon^2}{R_1^2}}|)] \tag{35}$$

$$Q_2(T,V) = V^2[1+\frac{3Gm^2\alpha(\frac{\epsilon}{R_1})}{2T(n)^{-\frac{1}{3}}}] \tag{36}$$

where

$$\alpha(\frac{\epsilon}{R_1}) = \sqrt[2]{1+(\frac{\epsilon}{R_1})^2}+(\frac{\epsilon}{R_1})^2 ln\frac{\frac{\epsilon}{R_1}}{1+\sqrt[2]{1+(\frac{\epsilon}{R_1})^2}} \tag{37}$$

R_1 is the radius of the cell and $\bar{n}$ is the average number density per unit volume so that $(\bar{n})^{\frac{-1}{3}} \approx \bar{r}$. Considering scale transformation for temperature T and number density $\bar{n}$.[3].

$$T \to \lambda^{-1}T \tag{38}$$

$$\bar{n} \to \lambda^{-3}\bar{n} \tag{39}$$

The factor $\frac{1}{T}(\bar{n})$ will thus be invariant

$$\frac{1}{T(\bar{n})^{-\frac{1}{3}}} \to \bar{n}T^{-3} \tag{40}$$

and to ensure $\frac{Gm^2}{T(\bar{n})^{-\frac{1}{3}}}$ is dimensionless, the ratio has to be transformed:

$$\frac{Gm^2}{T\bar{n}^{-\frac{1}{3}}} \to (Gm^2)^3\bar{n}T^{-3} \tag{41}$$

With scale transformation, the $N = 2$ configuration integral will have the form:

$$Q_2(T,V) = V^2[1 + \beta\bar{n}T^{-3}\alpha(\frac{\epsilon}{R_1})] \tag{42}$$

where $\beta = (\frac{3}{2})(Gm^2)^3$. The process is repeated for higher order of N and $Q_3(T,V), Q_4(T,V)$ and $Q_N(T,V)$ are :

$$Q_3(T,V) = \int\int\int (1 + f_{12})(1 + f_{23})d^3r_1 d^3r_2 d^3r_3 \tag{43}$$

$$Q_3(T,V) = \int\int\int [1 + \frac{Gm^2}{T({r_{23}}^2 + \epsilon^2)^{\frac{1}{2}}}][1 + \frac{Gm^2}{T({r_{12}}^2 + \epsilon^2)^{\frac{1}{2}}}]d^3r_1 d^3r_2 d^3r_3 \tag{44}$$

The two parts can be solved individually in the same way like Q_2

$$Q_3(T,V) = V^3[1 + \beta\bar{n}T^{-3}\alpha(\frac{\epsilon}{R_1})]^2 \tag{45}$$

$$Q_4(T,V) = V^4[1 + \beta\bar{n}T^{-3}\alpha(\frac{\epsilon}{R_1})]^3 \tag{46}$$

In general :

$$Q_N(T,V) = V^N[1 + \beta\bar{n}T^{-3}\alpha(\frac{\epsilon}{R_1})]^{N-1} \tag{47}$$

Subsequently, the configuration integral part is combined with the kinetic energy part to give the complete partition function:

$$Z_N(T,N) = \frac{1}{N!}(\frac{2\pi mT}{\lambda^2})^{\frac{3N}{2}}V^N[1 + \beta\bar{n}T^{-3}\alpha(\frac{\epsilon}{R_1})]^{N-1} \tag{48}$$

where $\beta = \frac{3}{2}G^3m^6$ and $\alpha(\frac{\epsilon}{R_1}) = \sqrt[2]{1 + (\frac{\epsilon}{R_1})^2} + (\frac{\epsilon}{R_1})^2 ln\frac{\frac{\epsilon}{R_1}}{1+\sqrt[2]{1+(\frac{\epsilon}{R_1})^2}}$ This is the partition function for gravitational clustering of galaxies. From the partition function, the equation of state and thermodynamic functions are calculated. The Helmholtz free energy can be calculated:

$$F = -TlnZ_N(T,V) = NTln(\frac{N}{V}T^{\frac{-3}{2}}) - NTln[1+\beta\bar{n}T^{-3}\alpha(\frac{\epsilon}{R_1})] - \frac{3}{2}NTln(\frac{2\pi m}{\lambda^2}) - NT \tag{49}$$

Here $N - 1$ has been approximated as N for large N and invoking Stirling's approximation where $lnN! = NlnN - N$. The other thermodynamics quantities are gotten from the free energy

$$S = -(\frac{\partial F}{\partial T}) \tag{50}$$

$$S = -Nln(\frac{N}{V}T^{\frac{-3}{2}}) + Nln[1 + \beta\bar{n}T^{-3}\alpha(\frac{\epsilon}{R_1})] + \frac{3}{2}Nln(\frac{2\pi m}{\lambda^2}) - 3Nb_\epsilon + \frac{5}{2}N \quad (51)$$

where b_ϵ is given by:

$$b_\epsilon = \frac{\beta\bar{n}T^{-3}\alpha(\frac{\epsilon}{R_1})}{1 + \beta\bar{n}T^{-3}\alpha(\frac{\epsilon}{R_1})} \quad (52)$$

$$U = F + TS = \frac{3}{2}NT(1 - 2b_\epsilon) \quad (53)$$

$$P = -(\frac{\partial F}{\partial V}) = \frac{NT}{V}(1 - 2b_\epsilon) \quad (54)$$

where V is contained in the number density $\bar{n}$ where $\bar{n} = \frac{\bar{N}}{V}(\frac{u}{T}) = \frac{1}{T}(\frac{\partial F}{\partial N})$(55)

$$(\frac{u}{T}) = ln(\frac{N}{V}T^{\frac{-3}{2}}) - ln[1 + \beta\bar{n}T^{-3}\alpha(\frac{\epsilon}{R_1})] - \frac{3}{2}ln(\frac{2\pi m}{\lambda^2}) - b_\epsilon \quad (56)$$

The equation can be simplified to a form with b_ϵ.The following parts follows the attempt to change the equation to a form containing only thermodynamics quantities and b_ϵ

$$-ln[1 + \beta\bar{n}T^{-3}\alpha(\frac{\epsilon}{R_1})] = ln\frac{1}{[1 + \beta\bar{n}T^{-3}\alpha(\frac{\epsilon}{R_1})]} \quad (57)$$

The following parts follows the attempt to change the equation to a form containing only thermodynamics quantities and b_ϵ

$$ln\frac{1}{[1 + \beta\bar{n}T^{-3}\alpha(\frac{\epsilon}{R_1})]} = ln\frac{1 + [\beta\bar{n}T^{-3}\alpha(\frac{\epsilon}{R_1})] - [\beta\bar{n}T^{-3}\alpha(\frac{\epsilon}{R_1})]}{[1 + \beta\bar{n}T^{-3}\alpha(\frac{\epsilon}{R_1})]} \quad (58)$$

$$ln\frac{1 + [\beta\bar{n}T^{-3}\alpha(\frac{\epsilon}{R_1})] - [\beta\bar{n}T^{-3}\alpha(\frac{\epsilon}{R_1})]}{[1 + \beta\bar{n}T^{-3}\alpha(\frac{\epsilon}{R_1})]} = ln[1 - \frac{[\beta\bar{n}T^{-3}\alpha(\frac{\epsilon}{R_1})]}{1 + [\beta\bar{n}T^{-3}\alpha(\frac{\epsilon}{R_1})]}] \quad (59)$$

$$ln[1 - \frac{[\beta\bar{n}T^{-3}\alpha(\frac{\epsilon}{R_1})]}{1 + [\beta\bar{n}T^{-3}\alpha(\frac{\epsilon}{R_1})]}] = ln(1 - b_\epsilon) \quad (60)$$

$$(\frac{u}{T}) = ln(\frac{N}{V}T^{\frac{-3}{2}}) + ln(1 - b_\epsilon) - \frac{3}{2}ln(\frac{2\pi m}{\lambda^2}) - b_\epsilon \quad (61)$$

The $(\frac{u}{T})$ is used to formulate the grand canonical partition function.The clustering parameter relates to the clustering parameter for point mass b by:

$$b_\epsilon = \frac{b\alpha(\frac{\epsilon}{R_1})}{1 + b[\alpha(\frac{\epsilon}{R_1}) - 1]} \quad (62)$$

6.1 DISTRIBUTION FUNCTION

With the result from the partition function, the distribution function can be found. The clustering of the galaxies can be characterized by the full probability distribution function. The distribution function $f_V(N)$ represent the probability of finding N number of particles in a cell of fixed volume. Since particles and energy can be exchanged between cells, the grand canonical ensemble will be utilised. The grand canonical partition function is a weighted sum of all canonical partition function:

$$Z_G(T,V,z) = \sum_{N=0}^{\infty} e^{\frac{Nu}{T}} Z_N(T,V) \tag{63}$$

The distribution function which is the probability of finding N particles in a cell of volume in the grand canonical ensemble:

$$f(N) = \frac{\sum_{i=0}^{\infty} e^{\frac{Nu}{T}} e^{\frac{-U_i}{T}}}{Z_G(T,V,z)} \tag{64}$$

$$f(N) = \frac{e^{\frac{Nu}{T}} Z_N(T,V)}{Z_G(T,V,z)} \tag{65}$$

For point masses with $\alpha = 1$:

$$e^{\frac{Nu}{T}=(\frac{\bar{N}}{V}T^{\frac{-3}{2}})^N(1-b)^N e^{-Nb}(\frac{2\pi m}{\lambda^2})} \tag{66}$$

$$Z_N(T,N) = \frac{1}{N!}(\frac{2\pi m}{\lambda^2})(VT^{\frac{3}{2}})^N[1+\frac{Nb}{\bar{N}(1-b)}]^{N-1} \tag{67}$$

The grand canonical partition function Z_G is related to the thermodynamic variables by:

$$lnZ_G = \frac{PV}{T} = \bar{N}(1-b) \tag{68}$$

Thus distribution function can be obtained from by incorporating equations in (66) (67) into (65) :

$$f(N) = \frac{(\bar{N}(1-b))^N}{N!} e^{-Nb}[1+\frac{Nb}{\bar{N}(1-b)}]^{N-1}(Z_G)^{-1} \tag{69}$$

Utilising equation (68), the distribution function can be found to be:

$$lnf(N) = ln(\frac{(\bar{N}(1-b))^N}{N!} e^{-Nb}[1+\frac{Nb}{\bar{N}(1-b)}]^{N-1}(Z_G)^{-1}) \tag{70}$$

$$lnf(N) = ln(\frac{(\bar{N}(1-b))^N}{N!} e^{-Nb} [\frac{Nb + \bar{N}(1-b)}{\bar{N}(1-b)}]^{N-1}) + \bar{N}(1-b) \tag{71}$$

$$f(N) = e^{ln(\frac{(\bar{N}(1-b))^N}{N!} e^{-Nb} [\frac{Nb+\bar{N}(1-b)}{\bar{N}(1-b)}]^{N-1}) + \bar{N}(1-b)} \tag{72}$$

$$f(N) = \bar{N}(1-b)[\bar{N}(1-b) + Nb]^{N-1} e^{-\bar{N}(1-b) - Nb} \tag{73}$$

The distribution function is completed and the process repeated with non point mass:

$$e^{\frac{Nu}{T} = (\frac{\bar{N}}{V} T^{\frac{-3}{2}})^N (1-b_\epsilon)^N e^{-Nb_\epsilon} (\frac{2\pi m}{\lambda^2})} (74)$$

$$Z_N(T, N) = \frac{1}{N!} (\frac{2\pi m}{\lambda^2}) (VT^{\frac{3}{2}})^N [1 + \frac{Nb_\epsilon}{\bar{N}(1-b_\epsilon)}]^{N-1}] \tag{75}$$

$$lnZ_G = \Phi = \frac{PV}{T} = \bar{N}(1-b_\epsilon) \tag{76}$$

$$f(N) = \bar{N}(1-b_\epsilon)[\bar{N}(1-b_\epsilon) + Nb_\epsilon]^{N-1} e^{-\bar{N}(1-b_\epsilon) - Nb_\epsilon} \tag{77}$$

As ϵ tends to zero , the point mass distribution function is obtained. This is the distribution for gravitational clustering that will be used as a theoretical expected result for the anisotropy in cosmic microwave background.

6.2 MOMENTS AND TWO POINT CORRELATION FUNCTION

The moments of the distribution function is related to the moments of the distribution and the correlation function[2]. The correlation function can be measured from the moments of counts in cell distribution. The two point correlation function describes clustering and can be approximated with power law approximation by restricting to the small scales. The relation between of two point correlation function and the moments of the counts in cell distribution:

$$\langle(\Delta N)^2\rangle = \bar{N}^2 \bar{\epsilon}_2 + \bar{N} \tag{78}$$

where$\bar{N}$ is the mean number of galaxies in a cell and the volume integral of the N-point correlation function

$$\bar{\epsilon}_2 = \frac{\langle(\Delta N)^2\rangle - \bar{N}}{\bar{N}^2} \tag{79}$$

This allows for a method to determine two point correlation function from a series of measurements of counts in cells distribution function.In this project the moment of this distribution is used extensively. The moment of distribution can be related to b and found from the counts in cell distribution. The grand canonical partition function can be related to mean and moment with these equation from statistical mechanics:[3]

$$\bar{N} = \frac{1}{\beta}\frac{\partial lnZ}{\partial u} \tag{80}$$

$$\langle(\Delta N)^2\rangle = \frac{1}{\beta^2}\frac{\partial^2 lnZ}{\partial u^2} \tag{81}$$

Where Z here is the grand canonical partition function Z_G. With equation (63) and that $Z_N(T,V)$ is not dependent on u:

$$\bar{N} = \frac{1}{Z}\sum_i N_i e^{\frac{N_i u}{T}} Z_N(T,V) \tag{82}$$

$$\langle(\Delta N)^2\rangle = \sum_i [\frac{1}{Z}{N_i}^2 e^{\frac{N_i u}{T}} Z_N(T,V) - \frac{1}{Z^2}(N_i e^{\frac{N_i u}{T}} Z_N(T,V))^2] \tag{83}$$

$$\langle(\Delta N)^2\rangle = \sum_i [\frac{1}{Z}{N_i}^2 e^{\frac{N_i u}{T}} Z_N(T,V) - \bar{N}^2] \tag{84}$$

Thus by differentiating a variant of equation (82) with respect to u while keeping T and V constant:

$$\bar{N}Z = \sum_i \frac{N_i}{T} e^{\frac{N_i u}{T}} Z_N(T,V) \tag{85}$$

$$Z\frac{\partial \bar{N}}{\partial u} + \bar{N}\frac{\partial Z}{\partial u} = \sum_i \frac{N_i^2}{T} e^{\frac{N_i u}{T}} Z_N(T,V) \tag{86}$$

$$\frac{\partial \bar{N}}{\partial u} = \sum_i \frac{1}{Z}\frac{N_i^2}{T} e^{\frac{N_i u}{T}} Z_N(T,V) - \bar{N}\frac{1}{Z}\sum_i \frac{N_i}{T} e^{\frac{N_i u}{T}} Z_N(T,V) \tag{87}$$

$$T\frac{\partial \bar{N}}{\partial u} = \sum_i \frac{1}{Z} N_i^2 e^{\frac{N_i u}{T}} Z_N(T,V) - \bar{N}^2 \tag{88}$$

$$T\frac{\partial \bar{N}}{\partial u} = \langle(\Delta N)^2\rangle \tag{89}$$

Thus the mean and moment shown in form comprised of thermodynamic variable and the number fluctuations are given as:

$$\frac{\langle(\Delta N)^2\rangle}{\bar{N}^2} = (\frac{T}{\bar{N}}\frac{\partial \bar{N}}{\partial u})_{T,V} \tag{90}$$

with the Gibbs-Duhem relation $SdT - VdP + Ndu = 0$

$$-N^2(\frac{\partial u}{\partial N})_{T,V} = V^2(\frac{\partial P}{\partial V})_{T,N} \tag{91}$$

Thus the fluctuations will be given as:

$$\frac{\langle(\Delta N)^2\rangle}{\bar{N}^2} = -\frac{T}{V^2}\frac{\partial V}{\partial P} \tag{92}$$

$$\frac{\langle(\Delta N)^2\rangle}{\bar{N}^2} = \frac{T}{\bar{N}V}(\frac{\partial N}{\partial P})_{T,V} \tag{93}$$

By using equation(68),(93) and $x = b_0\bar{n}T^{-3}$ given in [5] with scale transformation such that $b(\bar{n},T) = b(\bar{n}T^{-3}) + x\frac{db}{dx}$:

$$\langle(\Delta N)^2\rangle = \frac{\bar{N}}{[1 - b - x\frac{db}{dx}]} \tag{94}$$

With $b = \frac{x}{1+x}$ an ansatz initially proposed by Saslaw and Hamilton [5] also obtained through statistical derivation of the GQED by Ahmad et al(2002)

$$\frac{db}{dx} = \frac{1}{1+x} - \frac{x}{(1+x)^2} \tag{95}$$

Thus incorporating equation (95) into equation (94)

$$\langle(\Delta N)^2\rangle = \frac{\bar{N}}{(1-b)^2} \tag{96}$$

From the counts in cell distribution function,the moments of distribution can be related to b[1]:

$$\langle(\Delta N)^2\rangle = \frac{\bar{N}}{(1-b_\epsilon)^2} \tag{97}$$

The moments can be extracted from the counts in cell distribution with standard moment definition and extracted from the data with the following equations:

$$mean = \frac{\sum(x_i - \bar{x})}{N} \tag{98}$$

$$m_2 = \frac{\sum(x_i - \bar{x})^2}{N} \tag{99}$$

Hence the moments and clustering parameter can be calculated from equation 97 and 98. These are important components that are required in the distribution function.

7 DATA

The sample used in this project is from Planck data release 2 Cosmic Microwave Background map. These composite maps are produced from the frequency detection bolometer which tabulated the result to form a temperature profile. This data was obtained from IRSA website which stores data products of infrared and submillimetre. The data is a composite map for an all sky survey and has already been processed to remove foreground contamination. Composite CMB are produced with four different methods and the SMICA Cosmic microwave background map was utilised. SMICA cosmic microwave background map is processed by Planck software that combines all Planck input channel linearly. The Cosmic Microwave background map have intensity set between $-30\mu K$ and $30\mu K$.

The sample is stored in a fits format file. Fits stands for flexible image transport system, is a digital format for transportation and storage of image or data file widely used in astronomy. Fits directory can be used in many computing language and this project is be done with C programming language. In the C language library, fits library can be accessed with cfitsio.h library. The sample is formatted in a fits file with three extensions an image, intensity table and lastly the mask. The intensity table labelled COMP-MAP will be used for the project. IT is formatted in 2 columns and 50331648 rows. The file is structured in this way by Healpix format with $nside = 2048$. Healpix stands for Hierarchical Equal Area isoLatitude Pixelisation of a sphere. In Healpix format, a sphere is divided into many equal area pieces with each of centre of these pieces falling on isolatitude lines on the sphere. Thus, each equal area pieces will be labelled with a number in the fits table. Each of the rows within the fits represents data associated with one of the equal area pieces. Nside of a healpix format represent the degree of pixelisation of the sample with higher Nside number resulting in in pieces with smaller area and thus higher resolution. Healpix format allows for hierarchical storage of data which allows for more efficient and compact data storage. In C language, Healpix library is under chealpix.h library. The position of each equal area pieces can be extracted with the Healpix library.

8 COUNTS IN CELL STRATEGY

The counts in cell strategy involve dividing the sample into cells and recording the data within each of these cells. In this project, the number of hotspots within a cell will be recorded and used for analysis. The cells will be sorted by the number of hotspots it contains and the moment is calculated. The cells are allowed to overlap each other and must cover the whole sample. Square cells are selected for ease of calculation and inclusion into the program. The regions of galactic latitude between$-20°$ and $20°$ will be excluded from the sampling process to remove foreground contamination from galactic plane.

The following parts illustrate structure of the program (Appendix A).The program first includes all the library required for the implementation with fits and healpix library. Next the fits sample data file from Planck mission is open with a fits library function. A text file is prepared to store the results obtained from the program. Next three dynamic arrays are prepared to store the data from the Planck composite cosmic microwave map data and the number of hotspots in a cell. The mean and variance of the cosmic microwave background map is calculated and recorded as TAvg and TVar. The project is done by setting up a two dimensional grid with axes labelled by θ and ϕ. The axes represent the position of the cells and the data from the composite map. The θ and ϕ are similar to latitude and longitude in map system. The cells have a diameter of $1°$ and are placed initially at $\theta = -89.5\phi = -179.5$. This is due to the healpix data storage format of the composite map. The healpix ring format in the composite map stores the data by latitude with θ from $[0, \pi]$ and ϕ varies from $[0, 2\pi]$. This position will have to be adjusted to standard Geographic coordinate system with θ from $[-\frac{\pi}{2}, \frac{\pi}{2}]$ and ϕ from $[-\pi, \pi]$. This allows for ease of processing as most transformation follow the standard terrestrial longitude and latitude system. The cell centre position are denoted as yc and xc respectively for cell centre's θ and ϕ. The cell centre will be varied from the starting point of $\theta = -89.5\phi = -179.5$ by 1 degree through ϕ till $\phi = 179.5$. Once ϕ value have been varied from -179.5 to 179.5, the program will move the theta or latitude value by 1 degree and repeats the process of varying ϕ. This continues till the whole CMB map has been covered by cells Next any cell that falls within the exclusion region of -20° and 20° will be removed. The position of each point on the Planck CMB composite map can be retrieved with the healpix function. θ and ϕ given initially in $[0, \pi]$ and $[0, 2\pi]$ will be converted to $[-\frac{\pi}{2}, \frac{\pi}{2}]$ and ϕ from $[-\pi, \pi]$. Next sinusoidal equal area projection is applied to the position obtained after the standard galactic coordinate system adjustment. Sinusoidal equal area projections are given by:

$$x = (\phi) cos\theta \tag{100}$$

$$y = \theta \tag{101}$$

Where can be x can be treated as longitude and y as the latitude of the map. This is done to ensure that the sampling process is fair with each cell covering an equal area while preserving information for the latitude axis. Each position of the data point will be checked if it falls within the cell. This is done by specifying the maximum and minimum range of the cell with the cell radius. Each position associated with intensity data sample is compared with this maximum and minimum range. If the position of a data point falls within a cell, the data point will be recorded and the intensity value will be compared with the mean value. If the intensity value is more than the mean value with standard deviation of 2, the program will register a hotspot within the cell. This process continues till all the position of the data sample has been compared with this cell centre and the program will restart the process with a new cell centre. The cell centre is shift from its previous position along the longitudinal line and repeated along multiple longitudinal lines. The number of hotspot in a cell will be tabulated and listed in a histogram. The sum of the number hotspots will be recorded and used to calculate the mean and variance The counts in cell distribution $f_V(N)$ is then obtained by taking the histogram of the number of hotspots within a cell.The flowchart(figure 4) below summarises the step taken in the sampling process in the counts in cells strategy

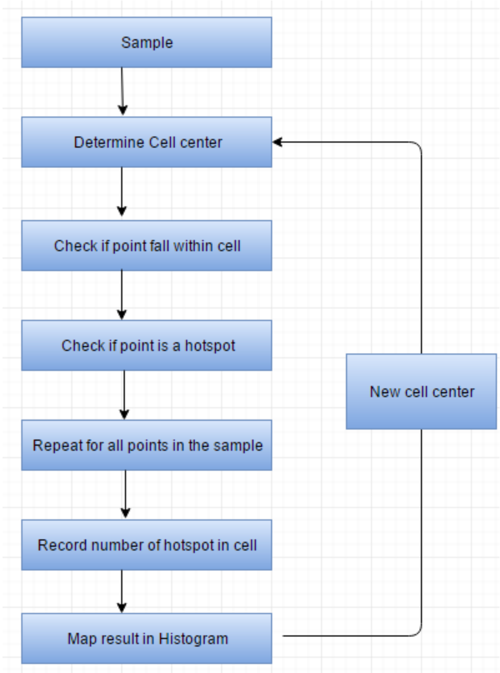

Figure 4: Flowchart for the method in counts in cell

9 RESULTS

The histogram is shown in the diagram below with cell length of 1° and total cell number of around 64 800: From the counts in cells distribution of the whole composite map, the

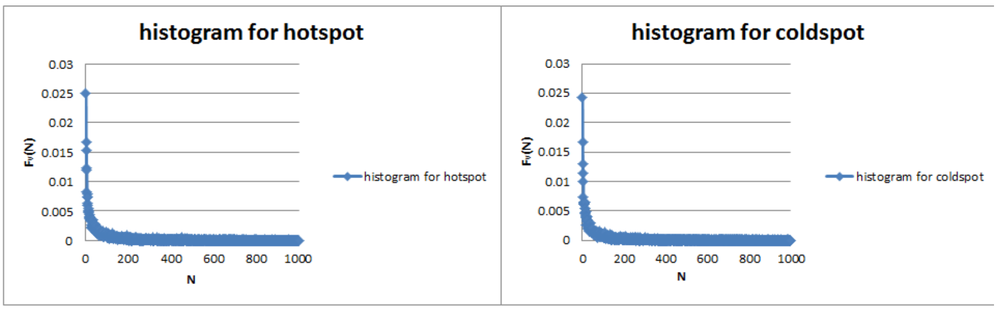

Figure 5: counts in cell with cell length 1° and number of cell 64800

distribution is heavily comprised of cells with zero hotspot. This may be due to the selection criteria in the program. The program rejects hotspot that is within 2 standard deviation of the temperature fluctuation. As a result based on Gaussian distribution which is expected of the fluctuations, only 5% of the fluctuations are reflected. This has the advantage of a shorter program runtime while highlighting the hotspot with higher energy which is directly related to the large scale structure of the universe. The mean and variance of the counts in cell distribution for hotspots of the whole composite map are 25.2 and 4824 respectively. The composite map can be also analysed by dividing the map into four sectors, each covering one quarter of the map. The quadrant is divided with the origin as the centre into 4 equal parts. This can be used to test for homogeneity across the sample. The results are seen in the figure below:

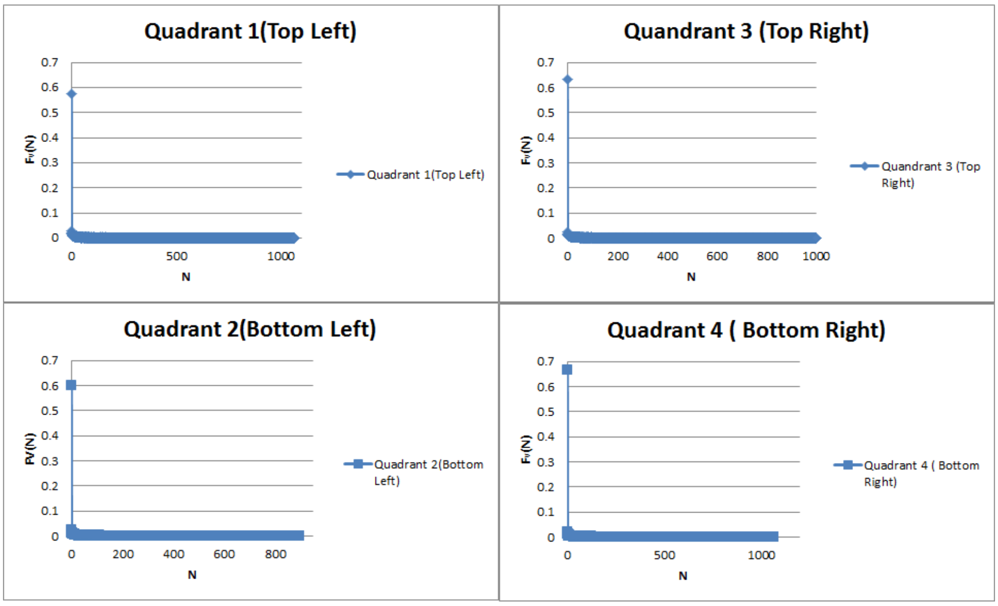

Figure 6: Quadrants with counts in cell with cell size 1° and number of cells 16200

The four quadrants agree to a certain extent with slight deviations across all four quadrants. The similar amount of variation suggest that this is the effect of cosmic variance.

The least-squares distance between the observed distribution and the theoretical distribution can be used as a measure of agreement between the two distribution. The least-squares distance is given by :

$$x = \sum_{N=0}^{N_{max}} (f_V(N)_{obs} - f_V(N))^2 \tag{102}$$

The mean and variance of the counts in cell distribution for hotspot was computed and b was obtained from equation(7). With the these values, the least square distance can be calculated. The project focuses on hotspot as they are readily comparable with the GQED distribution function. The result are shown in the table below:

Sample	Quadrant	Cells	$\bar{N}$	b	x
1	All	64800	30.6	0.994	0.0002
2	Quadrant 1	16200	29.4	0.994	0.03
3	Quadrant 2	16200	31.3	0.995	0.05
4	Quadrant 3	16200	30.3	0.995	0.08
5	Quadrant 4	16200	29.1	0.995	0.12

10 DISCUSSION AND CONCLUSION

The result from the sample can be compared with the least-square distance to check the validity of using gravitational clustering model for anisotropy in cosmic microwave background radiation. The anisotropy in cosmic microwave background radiation fits the model of gravitational clustering with GQED fairly well. The mean $\bar{N}$ found in this paper is much lower than that of counts in cell distribution of galaxy clustering with SDSS [2]. As mentioned before, this is due to the stringent selection process which accepts only 5% of the hotspots based on Gaussian distribution. However there may still be over counting of hotspot due to the project presenting a simplified procedure that assumes each point in the sample is independent. This results in the over-counting of the number of hotspot that may be multiple hotspots originating from one source. This can be deal with by implementing a checking system within the program. The program will check if a hotpot is independent with its surrounding spots. One method of doing so is to check if a hot spot is surrounded by other hotspot in a circle and if this is validated the whole sector will be taken as one large hotspot. Another method is to utilise the friends of friends algorithm is a percolation algorithm that can identify structure in galaxy distribution based on close proximity. The difference between the two methods is such that the first method excel at spotting circular hotspots while the latter are able to identify lop-sided hotspot.

The least square distance shows that the distribution function obtained in the experiment agree with the model distribution function obtained with gravitational Quasi-Equilibrium distribution. The results obtained in the project have a small percentage difference of around 10 % when compared to the GQED model. This supports the claim that anisotropy of cosmic microwave background radiation is caused by the mass fluctuations in the angular region tested in this project. The quadrants have generally similar distribution with slight variations of less than 10 % across the quadrants. The mean and x are similar across the quadrants. This point to anisotropy is homogeneous and Gaussian as expected. The slight variations may be a result of cosmic variance.

These are some factors to be considered for improving the accuracies of the project. The results may be inaccurate as the project is based on big bang cosmology and gravitational clustering. As cosmic microwave background radiation supports the notion of big bang cosmology and the project is centred on big bang cosmology. However, the initial conditions of the universe may be varied from our expectation. These are some speculative ideas in cosmology that may be responsible for anisotropy in cosmic microwave background radiation. The universe could be rotating and would induce anisotropy in the cosmic microwave background radiation. The surface of last scattering was rotating with respect to our local reference system, the equator would have a tangential velocity but the poles would not be rotating. Hence, there will be a relativistic redshift for Cosmic microwave background radiation and this will result in anisotropy in cosmic microwave background radiation at the equator.[3] Gravitational waves produced in the first instances of the early universe may cause anisotropy in cosmic microwave background radiation. Gravitational waves act as propagating fluctuations that would lead to fluctuations in density in the surface of last scattering. These fluctuations may be the cause of anisotropy instead of the adiabatic fluctuations.[3] Due to time constraints, the methodology of sample processing has been simplified. These features that can improve the results for a more accurate reading but extend the time required for the program. The project can be better verified with varying cell size and different methods for cell selection. This allows for a better test of homogeneity across the sample. A map projection for Mollweide projection can be used instead of sinusoidal equal area projection. An additional feature that can be improved upon is the test to check if each hotspot is independent. This feature will prevent over counting of the number of hotspot.

In conclusion, the counts in cell distribution of anisotropy in cosmic microwave background radiation agree with the Gravitational Quasi-Equilibrium distribution model up to a good extent. This supports the claim of anisotropy in cosmic microwave background radiation being strongly related to the sources for galaxy clustering specifically the mass fluctuations. This allows for the use of the composite map for cosmic microwave background radiation as a map of primordial universe. It allows for reference with other all sky survey of the universe to paint a better picture of universe.

11 Appendix A

11.1 C program code for whole composite map

```
#include <string.h>

#include <stdio.h>

#include <stdlib.h>

#include <cfitsio/fitsio.h>

#include <math.h>

#include <chealpix.h>

#include <math.h>


#define CELLSIZE 1.0

#define NSIGMA 2.0

int main()

{

   fitsfile *fptr;


   int status = 0, anynul, m, n, mmax, nmax, a, hotspot, coldspot;

   long nrows, jj,  b;

   double cellsize, theta, phi, xc, yc, x, cellradius;

   char filename[]  = "COM_CMB_IQU-smica-field-Int_2048_R2.01_full.fits"; /* name of

   double* array;

   int* mask;

   double TAvg = 0.0, TVar = 0.0;

   int startpixel = 0;

   int max_hotspot = 0, max_coldspot = 0;


   status = 0;
```

```
FILE *fp;
fp = fopen("results.txt","w");
if (fp == NULL) {
   printf("could not open.\n");
   exit(0);
}
fits_open_table(&fptr, filename, READONLY, &status);
if (status != 0) {
   printf("Could not open file\n");
   return -1;
}
fits_get_num_rows(fptr, &nrows, &status);
array = malloc(nrows * sizeof(double));
mask = malloc(nrows * sizeof(int));
for (jj = 1; jj <= nrows; jj++) {
   fits_read_col (fptr, TDOUBLE,1,jj, 1, 1, 0, &array[jj-1], &anynul, &status);
   fits_read_col (fptr, TINT,2,jj, 1, 1, 0, &mask[jj-1], &anynul, &status);
   TAvg += array[jj-1];
}
TAvg /= (double)nrows;
for (jj = 1; jj <= nrows; jj++) {
   TVar += pow(array[jj-1] - TAvg, 2.0);
}
TVar /= (double)nrows;

if (status != 0) {
   printf("Read error\n");
```

```
   return -1;
}
printf("File ok\n");
fflush(stdout);
cellsize = CELLSIZE*M_PI/180.0;
cellradius = cellsize/2;
mmax = 360.0/CELLSIZE;
nmax = 180.0/CELLSIZE;
for (n = 0; n < nmax; n++) {
   for (m = 0; m < mmax; m++) {
      yc = -M_PI/2 + (cellradius) + n*(cellsize);
      xc = -M_PI + (cellradius) + m*(cellsize);
      // skip(reject) out of bounds cell
         if ( (xc - cellradius) / cos(fabs(yc) + cellradius) < -M_PI ||
            (xc + cellradius) / cos(fabs(yc) + cellradius) > M_PI)
            continue;
         if (fabs(yc - cellradius) < -0.349 || fabs(yc + cellradius) > 0.349 )
            continue;

         hotspot = 0;
         coldspot = 0;
         for (a = startpixel; a < nrows ; a++)  {
            pix2ang_ring(2048, a, &theta, &phi);
            theta = theta - M_PI/2;
            phi = phi - M_PI;
            if (theta < yc - cellradius) {
               startpixel = a;
               continue;
```

```
            }
            if (theta > yc + cellradius) {
               break;
            }
            x = (phi) * cos( theta );
            ring2nest(2048, a, &b);
            if (theta > yc - cellradius && theta < yc + cellradius) {
               if(x > xc - cellradius && x < xc + cellradius) {
                  if(array[b] > TAvg + (NSIGMA*sqrt(TVar))) {
                     hotspot = hotspot + 1;
                  }
                  if(array[b] < TAvg - (NSIGMA*sqrt(TVar))) {
                     coldspot = coldspot + 1;
                  }
               }
            }
         }
         fprintf(fp, "%d\t%d\t%d\t%d\n", m, n, hotspot, coldspot);
         fflush(fp);
         if (max_hotspot < hotspot)
             max_hotspot = hotspot;
         if (max_coldspot < coldspot)
             max_coldspot = coldspot;
   }
}
printf("maximums: %d hotspot %d coldspot\n", max_hotspot, max_coldspot);
free(array);
free(mask);
```

```
   fclose(fp);
   fits_close_file(fptr, &status);
   if (status != 0) {
      printf("Could not close file\n");
      return -1;
   }

   return 0;
}
```

11.2 C program code for quadrant 1

```
#include <string.h>
#include <stdio.h>
#include <stdlib.h>
#include <cfitsio/fitsio.h>
#include <math.h>
#include <chealpix.h>
#include <math.h>

#define CELLSIZE 1.0
#define NSIGMA 2.0
int main()
{
   fitsfile *fptr;

   int status = 0, anynul, m, n, mmax, nmax, a, hotspot, coldspot;
```

```
long nrows, jj,  b;
double cellsize, theta, phi, xc, yc, x, cellradius;
char filename[]  = "COM_CMB_IQU-smica-field-Int_2048_R2.01_full.fits"; /* name of
double* array;
int* mask;
double TAvg = 0.0, TVar = 0.0;
int startpixel = 0;
int max_hotspot = 0, max_coldspot = 0;

status = 0;

FILE *fp;
fp = fopen("results.txt","w");
if (fp == NULL) {
   printf("could not open.\n");
   exit(0);
}
fits_open_table(&fptr, filename, READONLY, &status);
if (status != 0) {
   printf("Could not open file\n");
   return -1;
}
fits_get_num_rows(fptr, &nrows, &status);
array = malloc(nrows * sizeof(double));
mask = malloc(nrows * sizeof(int));
for (jj = 1; jj <= nrows; jj++) {
   fits_read_col (fptr, TDOUBLE,1,jj, 1, 1, 0, &array[jj-1], &anynul, &status);
   fits_read_col (fptr, TINT,2,jj, 1, 1, 0, &mask[jj-1], &anynul, &status);
```

```
   TAvg += array[jj-1];
}
TAvg /= (double)nrows;
for (jj = 1; jj <= nrows; jj++) {
   TVar += pow(array[jj-1] - TAvg, 2.0);
}
TVar /= (double)nrows;

if (status != 0) {
   printf("Read error\n");
   return -1;
}
printf("File ok\n");
fflush(stdout);
cellsize = CELLSIZE*M_PI/180.0;
cellradius = cellsize/2;
mmax = 180.0/CELLSIZE;
nmax = 90.0/CELLSIZE;
for (n = 0; n < nmax; n++) {
   for (m = 0; m < mmax; m++) {
      yc = -M_PI/2 + (cellradius) + n*(cellsize);
      xc = -M_PI + (cellradius) + m*(cellsize);
      // skip(reject) out of bounds cell
         if ( (xc - cellradius) / cos(fabs(yc) + cellradius) < -M_PI ||
            (xc + cellradius) / cos(fabs(yc) + cellradius) > M_PI)
            continue;
         if (fabs(yc - cellradius) < -0.349 || fabs(yc + cellradius) > 0.349 )
            continue;
```

```
hotspot = 0;
coldspot = 0;
for (a = startpixel; a < nrows ; a++)  {
   pix2ang_ring(2048, a, &theta, &phi);
   theta = theta - M_PI/2;
   phi = phi - M_PI;
   if (theta < yc - cellradius) {
      startpixel = a;
      continue;
   }
   if (theta > yc + cellradius) {
      break;
   }
   x = (phi) * cos( theta );
   ring2nest(2048, a, &b);
   if (theta > yc - cellradius && theta < yc + cellradius) {
      if(x > xc - cellradius && x < xc + cellradius) {
         if(array[b] > TAvg + (NSIGMA*sqrt(TVar))) {
            hotspot = hotspot + 1;
         }
         if(array[b] < TAvg - (NSIGMA*sqrt(TVar))) {
            coldspot = coldspot + 1;
         }
      }
   }
}
fprintf(fp, "%d\t%d\t%d\t%d\n", m, n, hotspot, coldspot);
```

```
            fflush(fp);
            if (max_hotspot < hotspot)
                max_hotspot = hotspot;
            if (max_coldspot < coldspot)
                max_coldspot = coldspot;
      }
   }
   printf("maximums: %d hotspot %d coldspot\n", max_hotspot, max_coldspot);
   free(array);
   free(mask);
   fclose(fp);
   fits_close_file(fptr, &status);
   if (status != 0) {
      printf("Could not close file\n");
      return -1;
   }

   return 0;
}
```

11.3 C program code for quadrant 2

```
#include <string.h>
#include <stdio.h>
#include <stdlib.h>
#include <cfitsio/fitsio.h>
#include <math.h>
```

```
#include <chealpix.h>
#include <math.h>

#define CELLSIZE 1.0
#define NSIGMA 2.0
int main()
{
   fitsfile *fptr;

   int status = 0, anynul, m, n, mmax, nmax, a, hotspot, coldspot;
   long nrows, jj,  b;
   double cellsize, theta, phi, xc, yc, x, cellradius;
   char filename[]  = "COM_CMB_IQU-smica-field-Int_2048_R2.01_full.fits"; /* name of
   double* array;
   int* mask;
   double TAvg = 0.0, TVar = 0.0;
   int startpixel = 0;
   int max_hotspot = 0, max_coldspot = 0;

   status = 0;

   FILE *fp;
   fp = fopen("results.txt","w");
   if (fp == NULL) {
      printf("could not open.\n");
      exit(0);
   }
   fits_open_table(&fptr, filename, READONLY, &status);
```

```
if (status != 0) {
   printf("Could not open file\n");
   return -1;
}
fits_get_num_rows(fptr, &nrows, &status);
array = malloc(nrows * sizeof(double));
mask = malloc(nrows * sizeof(int));
for (jj = 1; jj <= nrows; jj++) {
   fits_read_col (fptr, TDOUBLE,1,jj, 1, 1, 0, &array[jj-1], &anynul, &status);
   fits_read_col (fptr, TINT,2,jj, 1, 1, 0, &mask[jj-1], &anynul, &status);
   TAvg += array[jj-1];
}
TAvg /= (double)nrows;
for (jj = 1; jj <= nrows; jj++) {
   TVar += pow(array[jj-1] - TAvg, 2.0);
}
TVar /= (double)nrows;

if (status != 0) {
   printf("Read error\n");
   return -1;
}
printf("File ok\n");
fflush(stdout);
cellsize = CELLSIZE*M_PI/180.0;
cellradius = cellsize/2;
mmax = 360.0/CELLSIZE;
nmax = 90.0/CELLSIZE;
```

```
for (n = 0; n < nmax; n++) {
   for (m = 180; m < mmax; m++) {
      yc = -M_PI/2 + (cellradius) + n*(cellsize);
      xc = -M_PI + (cellradius) + m*(cellsize);
      // skip(reject) out of bounds cell
         if ( (xc - cellradius) / cos(fabs(yc) + cellradius) < -M_PI ||
            (xc + cellradius) / cos(fabs(yc) + cellradius) > M_PI)
            continue;
         if (fabs(yc - cellradius) < -0.349 || fabs(yc + cellradius) > 0.349 )
            continue;

         hotspot = 0;
         coldspot = 0;
         for (a = startpixel; a < nrows ; a++)  {
            pix2ang_ring(2048, a, &theta, &phi);
            theta = theta - M_PI/2;
            phi = phi - M_PI;
            if (theta < yc - cellradius) {
               startpixel = a;
               continue;
            }
            if (theta > yc + cellradius) {
               break;
            }
            x = (phi) * cos( theta );
            ring2nest(2048, a, &b);
            if (theta > yc - cellradius && theta < yc + cellradius) {
               if(x > xc - cellradius && x < xc + cellradius) {
```

```
                  if(array[b] > TAvg + (NSIGMA*sqrt(TVar))) {
                     hotspot = hotspot + 1;
                  }
                  if(array[b] < TAvg - (NSIGMA*sqrt(TVar))) {
                     coldspot = coldspot + 1;
                  }
               }
            }
         }
         fprintf(fp, "%d\t%d\t%d\t%d\n", m, n, hotspot, coldspot);
         fflush(fp);
         if (max_hotspot < hotspot)
             max_hotspot = hotspot;
         if (max_coldspot < coldspot)
             max_coldspot = coldspot;
   }
}
printf("maximums: %d hotspot %d coldspot\n", max_hotspot, max_coldspot);
free(array);
free(mask);
fclose(fp);
fits_close_file(fptr, &status);
if (status != 0) {
   printf("Could not close file\n");
   return -1;
}

return 0;
```

```
}
```

11.4 C program code for quadrant 3

```
#include <string.h>
#include <stdio.h>
#include <stdlib.h>
#include <cfitsio/fitsio.h>
#include <math.h>
#include <chealpix.h>
#include <math.h>

#define CELLSIZE 1.0
#define NSIGMA 2.0
int main()
{
   fitsfile *fptr;

   int status = 0, anynul, m, n, mmax, nmax, a, hotspot, coldspot;
   long nrows, jj,  b;
   double cellsize, theta, phi, xc, yc, x, cellradius;
   char filename[]  = "COM_CMB_IQU-smica-field-Int_2048_R2.01_full.fits"; /* name of
   double* array;
   int* mask;
   double TAvg = 0.0, TVar = 0.0;
   int startpixel = 0;
   int max_hotspot = 0, max_coldspot = 0;
```

```
status = 0;

FILE *fp;
fp = fopen("results.txt","w");
if (fp == NULL) {
   printf("could not open.\n");
   exit(0);
}
fits_open_table(&fptr, filename, READONLY, &status);
if (status != 0) {
   printf("Could not open file\n");
   return -1;
}
fits_get_num_rows(fptr, &nrows, &status);
array = malloc(nrows * sizeof(double));
mask = malloc(nrows * sizeof(int));
for (jj = 1; jj <= nrows; jj++) {
   fits_read_col (fptr, TDOUBLE,1,jj, 1, 1, 0, &array[jj-1], &anynul, &status);
   fits_read_col (fptr, TINT,2,jj, 1, 1, 0, &mask[jj-1], &anynul, &status);
   TAvg += array[jj-1];
}
TAvg /= (double)nrows;
for (jj = 1; jj <= nrows; jj++) {
   TVar += pow(array[jj-1] - TAvg, 2.0);
}
TVar /= (double)nrows;
```

```
if (status != 0) {
   printf("Read error\n");
   return -1;
}
printf("File ok\n");
fflush(stdout);
cellsize = CELLSIZE*M_PI/180.0;
cellradius = cellsize/2;
mmax = 180.0/CELLSIZE;
nmax = 180.0/CELLSIZE;
for (n = 90; n < nmax; n++) {
   for (m = 0; m < mmax; m++) {
      yc = -M_PI/2 + (cellradius) + n*(cellsize);
      xc = -M_PI + (cellradius) + m*(cellsize);
      // skip(reject) out of bounds cell
         if ( (xc - cellradius) / cos(fabs(yc) + cellradius) < -M_PI ||
            (xc + cellradius) / cos(fabs(yc) + cellradius) > M_PI)
            continue;
         if (fabs(yc - cellradius) < -0.349 || fabs(yc + cellradius) > 0.349 )
            continue;

         hotspot = 0;
         coldspot = 0;
         for (a = startpixel; a < nrows ; a++)  {
            pix2ang_ring(2048, a, &theta, &phi);
            theta = theta - M_PI/2;
            phi = phi - M_PI;
            if (theta < yc - cellradius) {
```

```
                startpixel = a;
                continue;
            }
            if (theta > yc + cellradius) {
                break;
            }
            x = (phi) * cos( theta );
            ring2nest(2048, a, &b);
            if (theta > yc - cellradius && theta < yc + cellradius) {
                if(x > xc - cellradius && x < xc + cellradius) {
                    if(array[b] > TAvg + (NSIGMA*sqrt(TVar))) {
                        hotspot = hotspot + 1;
                    }
                    if(array[b] < TAvg - (NSIGMA*sqrt(TVar))) {
                        coldspot = coldspot + 1;
                    }
                }
            }
        }
        fprintf(fp, "%d\t%d\t%d\t%d\n", m, n, hotspot, coldspot);
        fflush(fp);
        if (max_hotspot < hotspot)
             max_hotspot = hotspot;
        if (max_coldspot < coldspot)
             max_coldspot = coldspot;
    }
}
printf("maximums: %d hotspot %d coldspot\n", max_hotspot, max_coldspot);
```

```
   free(array);
   free(mask);
   fclose(fp);
   fits_close_file(fptr, &status);
   if (status != 0) {
      printf("Could not close file\n");
      return -1;
   }

   return 0;
}
```

11.5 C program code for quadrant 4

```
#include <string.h>
#include <stdio.h>
#include <stdlib.h>
#include <cfitsio/fitsio.h>
#include <math.h>
#include <chealpix.h>
#include <math.h>

#define CELLSIZE 1.0
#define NSIGMA 2.0
int main()
{
   fitsfile *fptr;
```

```
int status = 0, anynul, m, n, mmax, nmax, a, hotspot, coldspot;
long nrows, jj,  b;
double cellsize, theta, phi, xc, yc, x, cellradius;
char filename[]  = "COM_CMB_IQU-smica-field-Int_2048_R2.01_full.fits"; /* name of
double* array;
int* mask;
double TAvg = 0.0, TVar = 0.0;
int startpixel = 0;
int max_hotspot = 0, max_coldspot = 0;

status = 0;

FILE *fp;
fp = fopen("results.txt","w");
if (fp == NULL) {
   printf("could not open.\n");
   exit(0);
}
fits_open_table(&fptr, filename, READONLY, &status);
if (status != 0) {
   printf("Could not open file\n");
   return -1;
}
fits_get_num_rows(fptr, &nrows, &status);
array = malloc(nrows * sizeof(double));
mask = malloc(nrows * sizeof(int));
for (jj = 1; jj <= nrows; jj++) {
```

```
   fits_read_col (fptr, TDOUBLE,1,jj, 1, 1, 0, &array[jj-1], &anynul, &status);
   fits_read_col (fptr, TINT,2,jj, 1, 1, 0, &mask[jj-1], &anynul, &status);
   TAvg += array[jj-1];
}
TAvg /= (double)nrows;
for (jj = 1; jj <= nrows; jj++) {
   TVar += pow(array[jj-1] - TAvg, 2.0);
}
TVar /= (double)nrows;

if (status != 0) {
   printf("Read error\n");
   return -1;
}
printf("File ok\n");
fflush(stdout);
cellsize = CELLSIZE*M_PI/180.0;
cellradius = cellsize/2;
mmax = 360.0/CELLSIZE;
nmax = 180.0/CELLSIZE;
for (n = 90; n < nmax; n++) {
   for (m = 180; m < mmax; m++) {
      yc = -M_PI/2 + (cellradius) + n*(cellsize);
      xc = -M_PI + (cellradius) + m*(cellsize);
      // skip(reject) out of bounds cell
         if ( (xc - cellradius) / cos(fabs(yc) + cellradius) < -M_PI ||
            (xc + cellradius) / cos(fabs(yc) + cellradius) > M_PI)
            continue;
```

```
if (fabs(yc - cellradius) < -0.349 || fabs(yc + cellradius) > 0.349 )
   continue;

hotspot = 0;
coldspot = 0;
for (a = startpixel; a < nrows ; a++)  {
   pix2ang_ring(2048, a, &theta, &phi);
   theta = theta - M_PI/2;
   phi = phi - M_PI;
   if (theta < yc - cellradius) {
      startpixel = a;
      continue;
   }
   if (theta > yc + cellradius) {
      break;
   }
   x = (phi) * cos( theta );
   ring2nest(2048, a, &b);
   if (theta > yc - cellradius && theta < yc + cellradius) {
      if(x > xc - cellradius && x < xc + cellradius) {
         if(array[b] > TAvg + (NSIGMA*sqrt(TVar))) {
            hotspot = hotspot + 1;
         }
         if(array[b] < TAvg - (NSIGMA*sqrt(TVar))) {
            coldspot = coldspot + 1;
         }
      }
   }
```

```
            }
            fprintf(fp, "%d\t%d\t%d\t%d\n", m, n, hotspot, coldspot);
            fflush(fp);
            if (max_hotspot < hotspot)
                max_hotspot = hotspot;
            if (max_coldspot < coldspot)
                max_coldspot = coldspot;
        }
    }
    printf("maximums: %d hotspot %d coldspot\n", max_hotspot, max_coldspot);
    free(array);
    free(mask);
    fclose(fp);
    fits_close_file(fptr, &status);
    if (status != 0) {
        printf("Could not close file\n");
        return -1;
    }

    return 0;
}
```

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