NATIONAL UNIVERSITY OF SINGAPORE

PC3231 Electricity and Magnetism 2

(Semester I: AY 2008-09, 24, November)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **4** questions and comprises **4** printed pages.
2. Answer any **3** questions.
3. Answers to the questions are to be written in the answer books.
4. This is a CLOSED BOOK examination.
5. One Reference Sheet (A4 size, both sides) is allowed for this examination.
6. A Table of Constants is provided.

1. Boundary value problem

Consider a sphere of radius R made of homogeneous linear dielectric material. The general solution for the potential $V(r,\theta)$ may be expressed as a combination of Legendre polynomials:

$$V(r,\theta) = \sum_{0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$$

where

$$P_0(x) = 1$$
$$P_1(x) = x$$
$$P_2(x) = (3x^2 - 1)/2$$
$$P_3(x) = (5x^3 - 3x)/2$$

Suppose that the sphere is placed in an otherwise uniform external electric field $\mathbf{E_0}$ with magnitude E_0, show that the coefficients are

$$A_l = B_l = 0 \qquad for \quad l \neq 1$$
$$A_1 = -\frac{3}{\varepsilon_r + 2}E_0$$
$$B_1 = \frac{\varepsilon_r - 1}{\varepsilon_r + 2}R^3 E_0$$

Determine A_0 and B_0. Hence show that the field inside the sphere is uniform and is given by

$$\mathbf{E} = \frac{3}{\varepsilon_r + 2}\mathbf{E_0}$$

2. Propagation in a conductor

Consider a monochromatic plane wave impinging and traveling in a thick slab of conducting medium. The direction of propagation is perpendicular to the slab.

(i) Show that the electric and magnetic fields are no longer in phase.

(ii) Show that the energy is not equally shared between the electric and magnetic fields and that the magnetic contribution always dominates. Calculate the time-averaged energy density of the plane wave.

(iii) Show that the skin depth in a good conductor is $\lambda/2\pi$, where λ is the wavelength inside the conductor. Hence calculate the wavelength and skin depth in copper for radio waves at frequency 1 MHz. The conductivity σ for copper at room temperature is $\sigma = 6\mathrm{x}10^7$ in (ohm-meter)$^{-1}$.

3. TE mode in a rectangular waveguide

Consider the TE_{10} mode of a rectangular waveguide propagating in the z direction.

(i) What are the components (E_x, E_y) and (B_x, B_y, B_z) of the electric and magnetic fields for the TE_{10} mode? You are given that

$$E_x = \frac{i}{\left(\omega/c\right)^2 - k^2}\left(k\frac{\partial E_z}{\partial x} + \omega\frac{\partial B_z}{\partial y}\right)$$

$$E_y = \frac{i}{\left(\omega/c\right)^2 - k^2}\left(k\frac{\partial E_z}{\partial y} - \omega\frac{\partial B_z}{\partial x}\right)$$

$$E_z = 0$$

and

$$B_x = \frac{i}{\left(\omega/c\right)^2 - k^2}\left(k\frac{\partial B_z}{\partial x} - \frac{\omega}{c^2}\frac{\partial E_z}{\partial y}\right)$$

$$B_y = \frac{i}{\left(\omega/c\right)^2 - k^2}\left(k\frac{\partial B_z}{\partial y} + \frac{\omega}{c^2}\frac{\partial E_z}{\partial x}\right)$$

$$B_z = B_0 \cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}$$

(ii) Find the time averaged Poynting vector $\langle \mathbf{S} \rangle$ of the TE_{10} mode in the waveguide.

(iii) Consider a rectangular waveguide of cross-sectional dimensions 2.28 cm X 1.01 cm. If the driving frequency is $1.8\mathrm{X}10^{10}$ Hz, what TE modes will propagate in this waveguide?

4. Point charge in motion

The fields of a point charge q in arbitrary motion with velocity $\mathbf{v}$ and acceleration $\mathbf{a}$ are described by

$$\mathbf{E}(\mathbf{r},t) = \frac{q}{4\pi\varepsilon_0}\frac{\boldsymbol{\mathscr{r}}}{(\boldsymbol{\mathscr{r}}\cdot\mathbf{u})^3}\left[\left(c^2 - v^2\right)\mathbf{u} + \boldsymbol{\mathscr{r}}\times(\mathbf{u}\times\mathbf{a})\right]$$

where $\mathbf{u} = \hat{\boldsymbol{\mathscr{r}}}c - \mathbf{v}$ and $\boldsymbol{\mathscr{r}}$ the vector from the point charge to the observer,

and $\mathbf{B}(\mathbf{r},t) = \dfrac{1}{c}\hat{\boldsymbol{\mathscr{r}}}\times\mathbf{E}(\mathbf{r},t)$

(i) Explain with justifications why only the acceleration field contributes to the radiation. Hence obtain the Poynting vector $\mathbf{S}_{\mathrm{rad}}$ for the radiation field.

(ii) Show that for a point charge in slow motion $(v << c)$, the total power radiated P is given approximately by the Larmor formula:

$$P = \frac{\mu_0 q^2 \mathrm{a}^2}{6\pi \mathrm{c}}$$

(iii) Suppose an electron is decelerated at a constant rate *a* from an initial velocity v_0 down to zero. What fraction of its initial energy is lost to radiation? Assume $v_0 << c$ so that the Larmor formula can be used.

~ End of Examination Paper ~

TSH